

# Entropy Terms in Statistical Thermodynamic Analysis Formula for Non-stoichiometric Interstitial Compounds

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**Abstract** A series of statistical thermodynamic analyses were made since 1974 for different types of non-stoichiometric interstitial compounds  $\text{MX}_x$  under simplifying *a priori* assumption of constant interaction energy  $E(\text{X-X})$  between nearest neighbour interstitial atoms X within a homogeneity composition range of  $\text{MX}_x$  at arbitrary temperature  $T$  [K]. Mode of distribution of X atoms in interstitial sites in  $\text{MX}_x$  lattice is represented by number  $\theta$  of available interstitial sites for occupation by X atoms per M atom and the value of  $\theta$  is determined to fulfil the *a priori* assumption. Mode of atomic configuration would yield major contribution to entropy term  $\Delta S$  that appears in conventional thermodynamic expression of Gibbs free energy of formation,  $\Delta G$ , in form of  $T\Delta S$ . In the statistical thermodynamic formulation, contribution of tightly bound electron appearing in form of  $RT \ln f_x$  where  $f_x$  refers to atomic partition function of X atom in the  $\text{MX}_x$  lattice and  $R$  the universal gas constant. Judging from this mathematical form of the term,  $R \ln f_x$  is considered to represent entropic contribution from tightly bound electron to X atom in  $\text{MX}_x$  lattice. In the published series of works on statistical thermodynamic analysis for non-stoichiometric interstitial compounds, calculated values for  $R \ln f_x$  were reported but they were not reviewed with serious attention because  $R \ln f_x$  was considered merely as a secondary factor compared to principal factor  $E(\text{X-M})$  referring to interaction energy between X and M in  $\text{MX}_x$  lattice that represents enthalpy  $\Delta H$  in conventional thermodynamic term. In this review article, consideration is given exclusively to the factor  $R \ln f_x$  evaluated in statistical thermodynamic approach to non-stoichiometric interstitial compounds.

**Keywords** Entropy terms, Statistical thermodynamics, Interstitial compound, Non-stoichiometry

## 1. Introduction

Statistical thermodynamics is a unique analysis tool to span a bridge between **the experimentally measurable macroscopic thermodynamic state parameters** such as temperature  $T$ , equilibrium partial pressure  $p(\text{X}_2)$  of  $\text{X}_2$  gas and composition  $x$  in non-stoichiometric interstitial compound  $\text{MX}_x$  consisting of metal M and interstitial element X and **the non-measurable atomistic interaction energy parameters**  $E(i-j)$  ( $i, j = \text{M or X}$ ). Condensed phase  $\text{MX}_x$  subjected to statistical thermodynamic analysis might be in either solid or molten state.

Basic principles of statistical thermodynamics as well as practical analysis procedures are summarized in a classical text book authored by Fowler and Guggenheim [1].

In a monograph published in 2013 entitled "Statistical Thermodynamic Approach to Interstitial Non-stoichiometric Compounds (Hydride, Carbide, Nitride, Phosphide and

Sulphide)" [2], results of statistical thermodynamic analysis acquired on the basis of the simplifying *a priori* assumption of constant  $E(\text{X-X})$  within a homogeneity composition range of  $\text{MX}_x$  at arbitrary  $T$  were compiled [3-46] while a few more publications in this line of work arose thereafter [47-49]. In those publications, values for dissociation energy of  $\text{X}_2$  gas molecules,  $D(\text{X}_2)$  [ $\text{kJ}\cdot\text{mol}^{-1}$ ], characteristic temperature for rotation of  $\text{X}_2$  molecule,  $\Theta_r$  [K], and characteristic temperature for vibration of  $\text{X}_2$  molecule,  $\Theta_v$  [K], were taken from JANAF Thermochemical Tables [50] in the earlier works [3-43] or from its successor NIST-JANAF Thermochemical Tables [51] for the more recent calculations [44-49].

Metal constituent M in non-stoichiometric interstitial compound  $\text{MX}_x$  might be a single metal but might be binary substitutional alloy to be represented by  $\text{A}_{1-y}\text{B}_y$  or even multi-component alloy with number of metallic constituents being greater than 2. For such alloys represented by  $\text{A}_{1-y}\text{B}_y$ , statistical thermodynamic analysis results demonstrated that, depending on difference in affinity of X atom to constituent A and to constituent B, atom cluster might develop in the alloy lattice of  $\text{A}_{1-y}\text{B}_y\text{X}_x$  [9, 11, 16-19, 23, 25, 29, 33, 35-38, 40, 44, 46-49]. Statistical thermodynamic analysis was also made for alloy systems represented by  $\text{MZ}_z\text{X}_x$  in which Z is interstitial constituent whose affinity to M is stronger than

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that of X to M and composition  $z$  might be pre-determined arbitrarily before measuring equilibrium pressure - temperature - composition (*PTC*) relationships between  $MZ_xX_x$  and  $X_2$  gas [13, 31, 32, 34].

Literature information for *PTC* data source used in the cited statistical thermodynamic analysis works is not given in the text below as the *PTC* data sources used for the calculations were cited in References section in each publication.

## 2. Statistical Thermodynamic Analysis Procedure

### 2.1. Fundamental Equations

Generalized fundamental formulae proposed for this line of analysis of interstitial non-stoichiometric condensed phase  $MX_x$  are as follows [2, 3, 11, 12, 18, 20, 23, 25, 36, 37, 49].

$$A \equiv RT \ln \{[(\theta - x)/x] \{p(X_2)\}^{1/2}\} \\ = g(T) + \beta x E(X-X), \quad (1)$$

$$K(T) = g(T) - [D(X_2)/2 - RTC(T)] \\ = Q - RT \ln Zf_X(T), \quad (2)$$

$$C(T) = -(1/2) \ln \{[(4\pi m_X)^{3/2} k^{5/2}/h^3] \cdot \\ [(T^{7/2}/\Theta_r) \{1 + \Theta_r/(3T)\} \{\rho^2 v_0^*/2\} + \Theta_v/(4T) + \\ (1/2) \ln [1 - \exp(-\Theta_v/T)], \quad (3)$$

$$\ln f_X(T) = -\int_0^\infty g(v) \ln [1 - \exp(-hv/(kT))] dv + \ln \rho v_0, \quad (4)$$

$$Q + \beta x E(X-X) = \partial E(MX_x)/\partial n_X. \quad (5)$$

Symbols used in the above formulae are listed in the **APPENDIX** according to the classification of quality of the parameters.

In right hand side of Eq. (2), term  $Z$  is included explicitly in the second term,  $RT \ln Zf_X(T)$ . However, in the earlier publications on statistical thermodynamic analysis for non-stoichiometric interstitial alloy systems, explicit inclusion or not of this term was practised rather casually, except for analysis of very dilute interstitial alloy systems [11]. In reality,  $Z = 1$  is valid for many  $MX_x$  systems with  $x$  being close to 1. However, during the course of writing a monograph [2], it came to the author's mind that it is better to include the term  $Z$  in Eq. (2) explicitly. In fact, in the very first publication [3] in this line of work done for Cr sub-nitride,  $Cr_2N$ , formulation was made with  $Z = 2$  for  $\theta = 0.50$  ( $Z\theta = \theta = 1$  for octahedral sites (O-sites) occupation in hexagonal close-packed (*hcp*) lattice).

### 2.2. Partition Function

Procedure to derive fundamental equation (1) is reviewed briefly in the following.

Starting point of statistical thermodynamic analysis for non-stoichiometric interstitial alloy  $MX_x$  is the compilation of partition function  $PF(MX_x)$  for the condensed phase  $MX_x$

and  $PF(X_2)$  for ideal gas  $X_2$  at partial pressure  $p(X_2)$  being in equilibrium with  $MX_x$  at arbitrary  $T$ .

Formula for  $PF(X_2)$  at partial pressure  $p(X_2)$  and at temperature  $T$  is readily available in a classical text book authored by Fowler and Guggenheim [1]. On the other hand,  $PF(MX_x)$  has to be composed taking into account lattice structure of M and mode of distribution of X atoms over interstitial sites. Generalized formula for  $PF(MX_x)$  is represented by

$$PF(MX_x) = W(MX_x) f_M(T)^{n_M} f_X(T)^{n_X} \cdot \\ \exp[-E(MX_x)/(RT)], \quad (6)$$

where  $W(MX_x)$  refers to number of configurations of constituent atoms in  $MX_x$  lattice and  $E(MX_x)$  lattice energy of  $MX_x$ .

Expression for  $W(MX_x)$  is

$$W(MX_x) = Z^{n_X} \cdot (\theta \cdot n_M)! / [n_X! \cdot (\theta \cdot n_M - n_X)!]. \quad (7)$$

In most of  $MX_x$  compounds with non-ionic (metallic) characteristics ( $X = H, C, N, P$  or  $S$ ; but not  $O$ ), proportion of vacancies in M lattice is empirically acknowledged to be negligibly small even at elevated temperatures close to melting point. On the other hand, X atoms occupy interstitial sites (either octahedral interstitial sites (O-sites) or tetrahedral interstitial sites (T-sites)) according to mode of atomic distribution defined by the parameter  $\theta$ .

The term  $W(MX_x)$ , along with  $f_M$  and  $f_X$ , relates to entropy in conventional thermodynamic formulation as reviewed in some detail in [3].

By partial differentiation of  $PF(MX_x)$  with respect to number  $n_i$  of constituent  $i$  ( $i = M$  or  $X$ ), chemical potential  $\mu_i(MX_x)$  in  $MX_x$  is evaluated

$$\mu_i(MX_x) = -RT \cdot \{\partial [\ln PF(MX_x)] / \partial n_i\}. \quad (8)$$

As it is common to evaluate *PTC* relationship through isothermal experiment done for  $MX_x$  being in equilibrium with  $X_2$  gas at partial pressure  $p(X_2)$ , statistical thermodynamic analysis is made for  $i = X$  rather than for  $i = M$  using equilibrium relationship

$$\mu_X(MX_x) = \mu_X(X_2). \quad (9)$$

Expression for  $\mu_X(X_2)$  is readily drawn from the classical text book of Fowler and Guggenheim [1] and, as such, fundamental formula (1) is derived for statistical thermodynamic analysis of  $MX_x$  in equilibrium with  $X_2$  gas.

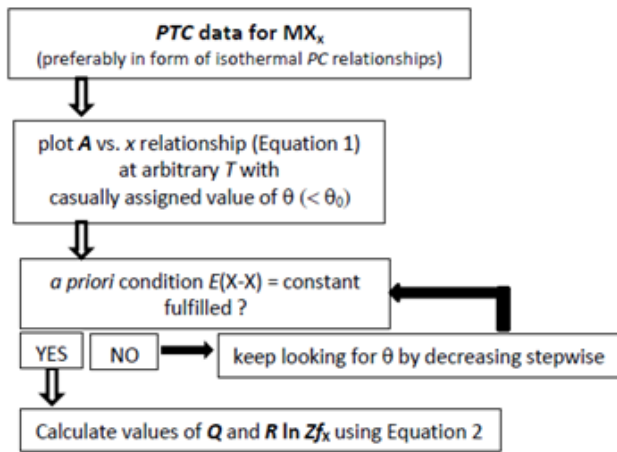
### 2.3. Analysis Procedure

At the onset of the analysis, isothermal  $A$  vs.  $x$  plots must be prepared from available isothermal *PC* relationship at arbitrary  $T$  using Eq. (1) by varying  $\theta$ . As understood from Eq. (1), slope of isothermal  $A$  vs.  $x$  plot would become proportional to  $E(X-X)$ . To fulfil the *a priori* assumption of constant  $E(X-X)$  within a homogeneity composition range of  $MX_x$  at arbitrary  $T$ ,  $\theta$  yielding linear  $A$  vs.  $x$  relationship over the entire homogeneity composition range of  $MX_x$  must be chosen for the subsequent calculations.

Then, from the intercept  $g(T)$  calculated using Eq. (1),

$K(T)$  vs.  $T$  relationship must be drawn using Eq. (2). Term  $Q$  on the right hand side in Eq. (2) refers to extent of stabilization of atom X in the  $\text{MX}_x$  lattice due to formation of X-M bonds in the  $\text{MX}_x$  lattice while the coefficient  $R \ln [Zf_x(T)]$  to  $T$  refers to electronic contribution to entropy term in thermodynamic sense. In fact, partition function  $f_x(T)$  of X atom in the  $\text{MX}_x$  lattice is a  $T$ -dependent factor as represented by Eq. (4) but, as the  $T$  range of statistical thermodynamic analysis for  $\text{MX}_x$  is typically no wider than 500 K, it has been a common practice to approximate  $f_x(T)$  as a  $T$ -independent constant factor [2-49].

For convenience of the readers, flow chart of the iterative determination procedure for a value of the parameter  $\theta$  is presented as Fig. 1.



**Figure 1.** Flow chart of the statistical thermodynamic analysis procedure accepting the a priori assumption of constant  $E(X-X)$  within a homogeneity composition range of  $\text{MX}_x$  at arbitrary  $T$ . (Reproduced from Fig. 1 in [49])

As represented by Eq. (5), term  $[Q + \beta x E(X-X)]$  refers to the net extent of stabilization of X atom in the  $\text{MX}_x$  lattice taking into account additionally the contribution  $\beta x E(X-X)$  from the X-X interaction besides  $Q$  which represents contribution of the X-M interaction. In Eq. (5),  $E(\text{MX}_x)$  refers to lattice energy of compound  $\text{MX}_x$  calculated taking into account all nearest neighbour pair-wise atomic interactions  $E(i-j)$  for all combinations of  $i$  and  $j$ .

For pragmatic convenience of calculating  $K(T)$  using Eq. (2),  $[D(X_2)/2 - RTC(T)]$  values for X = H and N are presented in tabulated form in [2] and [37] at 100 K interval from 0 K up to 3000 K so that  $[D(X_2)/2 - RTC(T)]$  value at arbitrary  $T$  is calculated readily by interpolation.

### 3. Values of the Term $R \ln Zf_x(T)$ in $\text{MX}_x$ Lattice Evaluated by Statistical Thermodynamics

Some representative calculation results for the term  $R \ln Zf_x(T)$  in  $\text{MX}_x$  reported in the past publications are reviewed hereafter classifying M into different categories.

#### 3.1. Estimated $R \ln Zf_x(T)$ Values under Approximation of Constant $f_x$ with $T$

In the standard statistical thermodynamic analysis carried out by the author,  $T$ -dependence of atomic partition function of  $f_x(T)$  was not taken into account and  $f_x(T)$  was assumed to hold  $T$ -independent constant value. For some  $\text{MH}_x$  systems in which values of vibration frequency  $\nu$  are available, attempts were made to evaluate  $R \ln f_x(T)$  as a function of  $T$ . The tentative calculation results for  $R \ln f_x(T)$  values as a function of  $T$  are reviewed later in the text in 3.2.

##### 3.1.1. Primary Solid Solutions $\text{MH}_x$

Values of  $R \ln Zf_H$  and other parameters evaluated for primary solid solutions  $\text{MH}_x$  are summarized in Table 1.

**Table 1.** Values of  $R \ln Zf_H$  and other parameters evaluated for primary solid solutions  $\text{MH}_x$

$\text{MH}_x$	$R \ln Zf_H$ [ $\text{J K}^{-1} \text{mol}^{-1}$ ]	$Q$ [ $\text{kJ mol}^{-1}$ ]	$\theta$	Ref.
$\text{ScH}_x$ (hcp)	72.1	-252	1.0	8
$\text{BaH}_x$ (hcp)	76.0	-249	1.5	8
$\text{TiH}_x$ (bcc)	33.2	-254	1.25	11
$\text{ZrH}_x$ (bcc)	33.6	-265	1.25	11
$\text{VH}_x$ (bcc)	64.7	-224	0.55	8
$\text{NbH}_x$ (bcc)	61.8	-230	0.75	8
$\text{TaH}_x$ (bcc)	62.4	-229	0.55	8
$\text{GdH}_x$ (hcp)	49.4	-259	1.0	7
$\text{TbH}_x$ (hcp)	43.8	-267	1.0	7
$\text{DyH}_x$ (hcp)	45.6	-266	1.0	7
$\text{LuH}_x$ (hcp)	39.7	-276	1.0	7

hcp: hexagonal close-packed crystal lattice.

bcc: body centered cubic crystal lattice.

It is intriguing to note that  $R \ln Zf_H$  values as well as  $Q$  values in  $\text{MH}_x$  for metals in the same group in the Periodic Table of the Elements are comparable to each other even when  $\theta$  values chosen to fulfil the a priori assumption for constant  $E(\text{H-H})$  within the given phase at arbitrary  $T$  were not the same; e.g.  $\theta = 1.0$  for  $\text{ScH}_x$  and  $\theta = 1.5$  for  $\text{BaH}_x$  (IIa-group metals) and  $\theta = 0.55$  for  $\text{VH}_x$  and  $\text{TaH}_x$  while  $\theta = 0.75$  for  $\text{NbH}_x$  (Va-group metals).

**Table 2.** Expression for  $Q$  for representative crystal lattice structures. (Reproduced from Table 1 in [18])

Crystal structure of $\text{MX}_x$	Expression for $Q^*$	
	O-site occupation	T-site occupation
bcc	$2E(\text{X-M})^{(I)} + 4E(\text{X-M})^{(II)} - E(\text{M-M})^{(II)}$	$4E(\text{X-M})^{(I)}$
fcc or hcp	$6E(\text{X-M})$	$4E(\text{X-M})$

fcc: face centered cubic crystal lattice. \* Super-scripts, (I) and (II), to  $E(\text{X-M})$  and  $E(\text{M-M})$  for bcc structure refer to first- and second-nearest neighbours, respectively.

Similarity in the values for  $R \ln Zf_H$  and  $Q$ , respectively, for  $\text{MH}_x$  of metals belonging to the same group in the Periodic Table appeared realistic supporting indirectly the

validity of the proposed statistical thermodynamic analysis procedure.

As presented in Table 2, expression for  $Q$  varies depending on whether the X atoms in  $MX_x$  lattice are distributed over O-sites or T-sites as well as on whether the crystal lattice structure is close-packed (*fcc* or *hcp*) or open (*bcc*). It must be noted that, in relatively open *bcc* lattice, contribution from second-nearest neighbour would enter into the expression for interaction energy term  $Q$ .

### 3.1.2. Hyper-stoichiometric Di-hydrides $MH_{2+\delta}$ of Lanthanides and Actinide

Analyses for hyper-stoichiometric di-hydrides  $MH_{2+\delta}$  of lanthanides (La, Ce, Pr, Nd, Sm) and actinides (Np, Pu, Am) with *fcc* lattice structure were made under assumption that H atoms up to  $H/M = 2.0$  occupy the T-sites and H atoms exceeding  $H/M = 2.0$  occupy randomly the O-sites [6]. Accepting this model,  $PF(MH_{2+\delta})$  is represented by (in the original paper [6], the hyper-stoichiometric phase is represented by  $MH_{2+x}$  instead of  $MH_{2+\delta}$ )

$$PF(MH_{2+\delta}) = \{n_M! / [(n_H - 2n_M)!(3n_M - n_H)!]\} \cdot f_M^{nM} (f_H^O)^{nH - 2nM} (f_H^T)^{2nM} \exp[-E(MH_{2+\delta}) / (RT)], \quad (10)$$

where  $f_H^O$  refers to atomic partition function of H atoms distributed over O-sites,  $f_H^T$  that of H atoms distributed over T-sites and  $E(MH_{2+\delta})$  lattice energy of  $MH_{2+\delta}$ .

Using this expression for  $PF(MH_{2+\delta})$ , expression for chemical potential  $\mu_H(MH_{2+\delta})$  of H in  $MH_{2+\delta}$  crystal lattice is represented by

$$\mu_H(MH_{2+\delta}) = RT \ln [\delta / (1 - \delta)] - RT \ln f_H^O + 6E(H-M)^O + 8E(H-H)^{O-T} + 12\delta E(H-H)^{O-O}, \quad (11)$$

where  $E(H-M)^O$  refers to interaction energy between the H atom in O-site and the M atom,  $E(H-H)^{O-T}$  interaction energy between the H atom in O-site and the H atom in T-site and  $E(H-H)^{O-O}$  interaction energy between the nearest neighbour H atoms in O-sites.

**Table 3.** Values of  $R \ln f_H^O$  and other parameters evaluated for hyper-stoichiometric  $MH_{2+\delta}$ . (Reproduced from Table 1 in [6])

$MH_{2+\delta}$ ( <i>fcc</i> )	$R \ln Zf_H^O$ [J·K <sup>-1</sup> ·mol <sup>-1</sup> ]	$Q_1$ [kJ·mol <sup>-1</sup> ]	$Q_2$ [kJ·mol <sup>-1</sup> ]	$D(M)$ [kJ·mol <sup>-1</sup> ]
LaH <sub>2.6</sub>	45.4	-249.5	-1028	431
CeH <sub>2.6</sub>	69.1	-236.7	-1021	423
PrH <sub>2.6</sub>	59.1	-241.9	-993	360
NdH <sub>2.6</sub>	57.0	-242.6	-919	326
SmH <sub>2.6</sub>	54.3	-239.4	-----	-----
NpH <sub>2.6</sub>	45.9	-255.1	-965	469
PuH <sub>2.6</sub>	43.3	-256.9	-----	-----
AmH <sub>2.6</sub>	53.5	-244.8	-842	276

$$Q_1 = 6E(H-M)^O + 8E(H-H)^{O-T}$$

$$Q_2 = 6E(M-M) + 8E(H-M)^T + 6E(H-H)^{T-T}$$

$$D(M) \approx -6E(M-M)^O$$

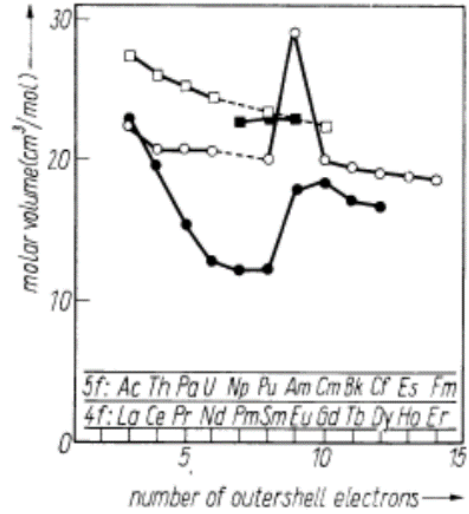
$E(M-M)$ : nearest neighbour M-M interaction energy in  $MH_{2+\delta}$ .

$E(H-M)^T$ : interaction energy between H atom in T-site and M atom.

$E(H-H)^{T-T}$ : interaction energy between nearest neighbour H atoms in T-sites.

$D_M$ : dissociation energy of pure metal M.

$E(M-M)^O$ : nearest neighbour M-M interaction energy in pure metal M.



**Figure 2.** Molar volume of some lanthanides (4*f*-group elements), actinides (5*f*-group elements) and their hydrides; M [52]: (○) 4*f*, (●) 5*f*.  $MH_{2.0}$  [53]: (□) 4*f*, (■) 5*f*. (Reproduced from Fig. 5 in [6])

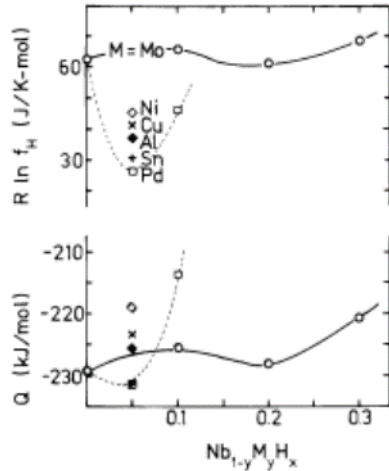
In Table 3, values of  $Q_2$  estimated from consideration for  $\mu_M(MH_{2+\delta})$  are listed besides  $Q_1$  evaluated from consideration for  $\mu_H(MH_{2+\delta})$ . Unlike similarities in values of  $R \ln Zf_H^O$  and  $Q_1$  among lanthanides (4*f*-group elements) and among actinides (5*f*-group elements), values of  $Q_2$  vary appreciably among lanthanides and among actinides. This aspect with  $Q_2$  was appreciated in terms of considerable extent of variation of molar volume of M [52] and  $MH_{2.0}$  [53] of 4*f*- and 5*f*-elements with respect to number  $m$  of the outer-shell *f*-electrons as represented in Fig. 2 and resultant variations of dissociation energy  $D(M)$  of these *f*-group metals with respect to number of the outer-shell *f*-electrons as listed in Table 3 [6].

### 3.1.3. Suppressed H Solubility for Va-group Alloys $A_{1-y}M_yH_x$ (A = V, Nb or Ta) by Alloying with Substitutional Metallic Constituent M (= Al, Co, Cr, Cu, Fe, Mo, Ni, Pd, Sn, Ru or W)

Statistical thermodynamic evaluations for parameters,  $\theta$ ,  $Q$  and  $R \ln Zf_H$ , were made for  $A_{1-y}B_yH_x$  alloys of Va-group metals (A = V, Nb or Ta) through alloying with substitutional metallic constituents M (= Al, Co, Cr, Cu, Fe, Mo, Ni, Pd, Sn, Ru or W). The evaluation results are listed in Table 4 [49]. Graphical presentation of values for  $R \ln Zf_H$  and  $Q$  for  $Nb_{1-y}M_yH_x$  alloys with suppressed H absorption from that in Nb through alloying with M (= Pd, Sn, Al, Cu, Ni or Mo) are reproduced in Fig. 3 [9].

It is evident in Table 4 that value of  $\theta$  for *bcc*  $A_{1-y}M_yH_x$  in which solubility of H is suppressed from that in  $AH_x$  became smaller than that for  $AH_x$ . However, it is noticed in Table 4 that, for some  $A_{1-y}M_yH_x$  in which H solubility was suppressed from that in  $AH_x$ ,  $Q(A_{1-y}M_yH_x)$  became more negative than  $Q(AH_x)$  meaning H in such  $A_{1-y}M_yH_x$  lattice was more stable with regard to interaction energy between H atom and metal atoms in  $A_{1-y}M_y$  lattice than H in  $AH_x$  in spite of the suppressed extent of H solubility; e.g.  $V_{0.95}Fe_{0.05}H_x$ ,  $V_{0.948}Co_{0.052}H_x$ ,  $Nb_{0.95}W_{0.05}H_x$ ,  $Nb_{0.95}Sn_{0.05}H_x$  and

$\text{Nb}_{0.95}\text{Pd}_{0.05}\text{H}_x$  in Table 4 for which the  $Q$  values are presented in bold letters. It is intriguing to notice in Table 4 that  $R \ln Zf_{\text{H}}$  values for these alloys with suppressed H solubility are relatively small compared with those for the other alloys with suppressed H solubility listed in this table.



**Figure 3.** Values of  $R \ln Zf_{\text{H}}$  and  $Q$  plotted as a function of  $y$  in bcc  $\text{Nb}_{1-y}\text{M}_y\text{H}_x$  ( $M = \text{Pd}, \text{Sn}, \text{Al}, \text{Cu}, \text{Ni}$  or  $\text{Mo}$ ). (Reproduced from Fig. 4 in [9])

**Table 4.** Available statistical thermodynamic interaction parameter values for bcc  $\text{A}_{1-y}\text{M}_y\text{H}_x$  that showed suppressed H solubility compared to that in bcc  $\text{AH}_x$ , where A refers to Va-group metals (V, Nb or Ta) (Reproduced from Table 2 in [49])

<i>bcc</i> $\text{A}_{1-y}\text{M}_y\text{H}_x$	$\theta$	$Q$ [kJ·mol <sup>-1</sup> ]	$R \ln Zf_{\text{H}}$ [J·K <sup>-1</sup> ·mol <sup>-1</sup> ]
$\text{VH}_x$	0.55	-223.6	64.7
$\text{V}_{0.96}\text{Cr}_{0.04}\text{H}_x$	0.525	-220.0	26.9
$\text{V}_{0.916}\text{Cr}_{0.084}\text{H}_x$	0.475	-207.2	54.5
$\text{V}_{0.949}\text{Mo}_{0.051}\text{H}_x$	0.525	-207.3	37.5
$\text{V}_{0.95}\text{Fe}_{0.05}\text{H}_x$	0.45	-229.4	23.7
$\text{V}_{0.948}\text{Co}_{0.052}\text{H}_x$	0.45	-230.1	21.5
$\text{NbH}_x$	0.75	-229.5	61.7
$\text{Nb}_{0.95}\text{Ru}_{0.05}\text{H}_x$	0.55	-221.8	41.2
$\text{Nb}_{0.95}\text{W}_{0.05}\text{H}_x$	0.725	-243.1	14.1
$\text{Nb}_{0.90}\text{Mo}_{0.10}\text{H}_x$	0.45	-225.5	65.4
$\text{Nb}_{0.80}\text{Mo}_{0.20}\text{H}_x$	0.30	-228.2	61.3
$\text{Nb}_{0.70}\text{Mo}_{0.30}\text{H}_x$	0.20	-220.8	67.7
$\text{Nb}_{0.95}\text{Al}_{0.05}\text{H}_x$	0.60	-225.8	36.9
$\text{Nb}_{0.95}\text{Cu}_{0.05}\text{H}_x$	0.60	-223.5	41.2
$\text{Nb}_{0.95}\text{Sn}_{0.05}\text{H}_x$	0.60	-231.8	30.6
$\text{Nb}_{0.95}\text{Ni}_{0.05}\text{H}_x$	0.60	-219.2	45.0
$\text{Nb}_{0.95}\text{Pd}_{0.05}\text{H}_x$	0.60	-231.8	26.1
$\text{Nb}_{0.90}\text{Pd}_{0.10}\text{H}_x$	0.45	-213.9	45.5
$\text{TaH}_x$	0.55	-229.4	62.4
$\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x$	0.55	-228.6	32.1

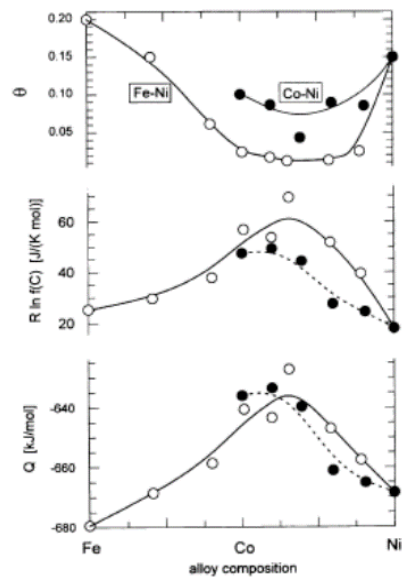
$Q$  values for  $\text{A}_{1-y}\text{M}_y\text{H}_x$  that were evaluated to be more negative than that for  $\text{AH}_x$  are displayed with bold letter.

In Table 4, unique situation is encountered for  $\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x$  in which H solubility was suppressed compared for  $\text{TaH}_x$  but  $\theta(\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x)$  was 0.55 holding

equal to  $\theta(\text{TaH}_x)$  and  $Q(\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x)$  was comparable to  $Q(\text{TaH}_x)$  while  $R \ln f_{\text{H}}(\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x) < R \ln f_{\text{H}}(\text{TaH}_x)$ .  $^{74}\text{W}$  is neighbouring element to  $^{73}\text{Ta}$  in the Periodic Table of the Elements and the outer-shell electron structure of  $^{74}\text{W}$ ,  $5d^46s^2$ , is quite similar to that of  $^{73}\text{Ta}$ ,  $5d^36s^2$ , and this fact might have led to similar values for  $\theta$  and  $Q$  between  $\text{TaH}_x$  and  $\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x$ , while electronic surrounding around H atom was modulated somewhat from  $\text{TaH}_x$  to  $\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x$  to lead to appreciable change in  $R \ln f_{\text{H}}$  from  $\text{TaH}_x$  (62.4 [J·K<sup>-1</sup>·mol<sup>-1</sup>]) to  $\text{Ta}_{0.95}\text{W}_{0.05}\text{H}_x$  (32.1 [J·K<sup>-1</sup>·mol<sup>-1</sup>]).

Values of  $Q$  and  $R \ln Zf_{\text{H}}$  evaluated for  $\text{Nb}_{0.95}\text{M}_{0.05}\text{H}_x$  ( $M = \text{Pd}, \text{Sn}, \text{Al}, \text{Cu}$  or  $\text{Ni}$ ) are plotted in Fig. 3 (in the original publication [9],  $R \ln Zf_{\text{H}}$  axis was indicated as  $R \ln f_{\text{H}}$  as seen above). It is seen in Fig. 3 that, at a fixed  $y = 0.05$  for  $\text{Nb}_{1-y}\text{M}_y\text{H}_x$ ,  $R \ln Zf_{\text{H}}(\text{Nb}_{0.95}\text{M}_{0.05}\text{H}_x)$  as well as  $Q(\text{Nb}_{0.95}\text{M}_{0.05}\text{H}_x)$  and  $\theta(\text{Nb}_{0.95}\text{M}_{0.05}\text{H}_x)$  (see Table 4) varied appreciably depending on the alloying constituent M reflecting difference in extent of modulation of electronic structure in the  $\text{Nb}_{0.95}\text{M}_{0.05}$  metal sub-lattice depending on the electronic structure around the alloyed M to Nb.

It is seen in Table 4 that  $\theta(\text{Nb}_{1-y}\text{Mo}_y\text{H}_x)$  decreased monotonically with increasing  $y$  from 0.75 at  $y = 0$ , through 0.45 at  $y = 0.10$  and 0.30 at  $y = 0.20$ , to 0.20 at  $y = 0.30$ . Nevertheless, variation patterns for  $R \ln Zf_{\text{H}}(\text{Nb}_{1-y}\text{Mo}_y\text{H}_x)$  and for  $Q(\text{Nb}_{1-y}\text{Mo}_y\text{H}_x)$  are not at all monotonical with respect to variation of  $y$  (Fig. 3). The same is true for  $\text{Nb}_{1-y}\text{Pd}_y\text{H}_x$ ;  $\theta$  decreased monotonically with increasing  $y$  from 0.75 at  $y = 0$  through 0.60 at  $y = 0.05$  to 0.45 at  $y = 0.10$  (Table 4) but patterns of variations of  $R \ln Zf_{\text{H}}(\text{Nb}_{1-y}\text{Pd}_y\text{H}_x)$  and  $Q(\text{Nb}_{1-y}\text{Pd}_y\text{H}_x)$  were not at all monotonical with respect to variation of  $y$  (Fig. 3).



**Figure 4.** Estimated values of  $\theta$ ,  $R \ln Zf_{\text{C}}$  and  $Q$  for *fcc*  $\text{Fe}_{1-y}\text{Ni}_y\text{C}_x$  and for *fcc*  $\text{Co}_{1-y}\text{Ni}_y\text{C}_x$  plotted as a function of alloy composition (mean number  $m$  of 3d electrons was 6 for Fe, 7 for Co and 8 for Ni). (Reproduced from Fig. 2 in [35])

Estimated values of  $\theta$ ,  $Q$  and  $R \ln Zf_{\text{C}}$  for *fcc*  $\text{Fe}_{1-y}\text{Ni}_y\text{H}_x$  and for *fcc*  $\text{Co}_{1-y}\text{Ni}_y\text{H}_x$  are plotted in Fig. 4 [35]. Composition axis of this plot corresponds to number of outer-shell 3d

electrons; mean number  $m$  of  $3d$  electrons being 6 for Fe, 7 for Co and 8 for Ni. Analysis for these alloys were done considering they were very dilute solutions of C in  $A_{1-y}B_y$  alloy lattice by taking an a priori assumption of

$$E(C-C) = 0, \quad (12)$$

within the given phase at arbitrary  $T$ .

As seen in Fig. 4, variation patterns of  $\theta$  for *fcc*  $Fe_{1-y}Ni_yH_x$  and for *fcc*  $Co_{1-y}Ni_yH_x$  with respect to  $m$  is not monotonical but it appears that, for both iron-group alloys,  $\theta$  value took minimum at around  $m \approx 7.25$  and, correspondingly, peak values for  $R \ln Z_{f_C}$  and  $Q$  emerged at around  $m \approx 7.25$ .

### 3.1.4. Influences of Concentration Range $x$ and Isotope Effect of Solute Element X in $MX_x$ ( $M = Ti$ or $Zr$ ; $X = H$ or $D$ ) on $R \ln Z_{f_X}$

Very dilute solution ranges of H and D (deuterium) in IVa-group metals, Ti and Zr, were analyzed under a priori assumption of

$$E(X-X) = 0, \quad (13)$$

at arbitrary  $T$  where X refers to H or D [11].

Analysis results are summarized in Table 5 in which results of analysis for  $MX_x$  in the moderate concentration range are also listed.

**Table 5.** Values of  $Q$  and  $R \ln Z_{f_X}$  for the very dilute ( $x < 0.015$ ) and the modest ( $0.1 < x < 0.8$ ) solute concentration ranges in  $MX_x$  ( $M = Ti$  or  $Zr$ ;  $X = H$  or  $D$ ). (Reproduced from Table 1 in [11])

$MX_x$	very dilute solute range		moderate solute range		$Z$
	$Q^d$ [kJ·mol <sup>-1</sup> ]	$R \ln Z_{f_X}^d$ [J·K <sup>-1</sup> ·mol <sup>-1</sup> ]	$Q$ [kJ·mol <sup>-1</sup> ]	$R \ln Z_{f_X}$ [J·K <sup>-1</sup> ·mol <sup>-1</sup> ]	
TiH <sub>x</sub>	-255.3 ± 1.4	60.5 ± 1.2	-253.5 ± 1.7	33.2 ± 1.4	≈ 27
TiD <sub>x</sub>	-265.7 ± 1.2	78.8 ± 1.1	-265.2 ± 1.8	50.4 ± 1.5	≈ 30
ZrH <sub>x</sub>	-249.9 ± 1.2	71.0 ± 1.0	-259.3 ± 2.2	33.6 ± 1.9	≈ 89
ZrD <sub>x</sub>	-273.4 ± 2.1	77.4 ± 1.8	-257.6 ± 3.7	61.6 ± 3.1	≈ 7

$Q^d$ :  $Q(MX_x)$  in the very dilute range of  $x$ ,  
 $f_X^d$ :  $f_X$  in the very dilute range of  $x$ .

On undertaking statistical thermodynamic analysis for  $MD_x$  being in equilibrium with  $D_2$  gas,  $C(T)$  for  $D_2$  gas had to be calculated using Eq.(3). On this calculation, common approximations were employed for estimating values of  $\Theta_r(D_2)$  and  $\Theta_v(D_2)$ , respectively, from available values of  $\Theta_r(H_2)$  and  $\Theta_v(H_2)$

$$\Theta_r(D_2) \approx \Theta_r(H_2)/2, \quad (14)$$

$$\Theta_v(D_2) \approx \Theta_v(H_2)/2^{1/2}. \quad (15)$$

Values  $D(D_2)$  of dissociation energy of  $D_2$  gas as a function of  $T$  were not available in JANAF Thermochemical Tables [50]. Hence, following approximation was used on the basis of the difference between  $D(D_2)$  at 0 K and  $D(H_2)$  at 0 K listed in [54] to estimate  $D(D_2)$  at arbitrary  $T$

$$D(D_2) \approx D(H_2) + 7.5 \text{ [kJ·mol}^{-1}\text{]}. \quad (16)$$

Further,  $\rho$  was taken to be 2 for H and 3 for D. On the basis of these approximations, analysis results summarized in Table 5 were obtained.

In [11], brief review was made on isotope effect of H and D in Ti lattice for atomic partition function expressed by Eq.(4). The essence of the review is summarized in the following.

Accepting the expression for  $\ln f_X(T)$  represented by Eq.(4), difference between  $\ln f_D$  and  $\ln f_H$  is given by

$$\begin{aligned} \ln f_D - \ln f_H &= \ln [f_D(T)/f_H(T)] = \\ &= \int_0^\infty \ln \{1 - \exp[-hv_H/(kT)]\} dv_H - \int_0^\infty \ln \{1 - \exp[-hv_D/(kT)]\} dv_D \\ &\quad + \ln \{3v_0(D)/[2v_0(H)]\}. \end{aligned} \quad (17)$$

Experimental data available on vibrational mode of H in M lattice were scarce [6, 8, 18, 55-57];  $v_H = 550 \text{ cm}^{-1}$  for H at O-sites in *fcc* Pd and  $v_H(1) = 970 \text{ cm}^{-1}$  (singlet) and  $v_H(2) = 1400 \text{ cm}^{-1}$  (doublet) for H at T-sites in *bcc* V, Nb or Ta.

Thus, in the earlier works [6, 8, 18], very crude tentative approximations were made to assume that the vibration frequency  $v_H$  in the metallic lattice was  $750 \text{ cm}^{-1}$  and that the multiplicity at this  $v_H$  was triple. Results of approximate calculations accepting this very crude vibrational mode for H in  $MH_x$  lattices are summarized in Table 6 as reviewed later in 3.2.

Accepting this crude model, Eq. (17) is reduced to

$$\begin{aligned} \ln (f_D/f_H) &= 3 \ln \{ [1 - \exp[-hv_H/(kT)]] / [1 - \exp[-hv_D/(kT)]] \} \\ &\quad + \ln \{ 3v_0(D)/[2v_0(H)] \}, \end{aligned} \quad (18)$$

where  $v_D$  might be estimated by approximating harmonic oscillator accepting  $v_H = 750 \text{ cm}^{-1}$

$$v_D = v_H/2^{1/2} \approx 530 \text{ cm}^{-1}. \quad (19)$$

Accepting this value for  $v_D$ , value of the ratio  $v_0(D)/v_0(H)$  was calculated to be 2.9 at  $T = 1200 \text{ K}$  implying that the statistical weight  $v_0(D)$  of D atom in  $TiD_x$  is different from  $v_0(H)$  of H atom in  $TiD_x$  which seemed realistic (isotope effect). It is also noticed in Table 3 that  $Q(TiD_x)$  and  $Q(TiH_x)$  in both the very dilute and the moderate solution concentrations were different from each other by the same degree of about  $10 \text{ [kJ·mol}^{-1}\text{]}$  from each other in either the very dilute and the modest solution concentration range (isotope effect).

Looking at Table 5, it is noticed that difference between  $R \ln Z_{f_D}^d$  ( $= 78.8 \text{ [J·K}^{-1}\text{·mol}^{-1}\text{]}$ ) and  $R \ln Z_{f_H}^d$  ( $= 60.5 \text{ [J·K}^{-1}\text{·mol}^{-1}\text{]}$ ) in the very dilute solution range and the difference between  $R \ln f_D$  ( $= 50.4 \text{ [J·K}^{-1}\text{·mol}^{-1}\text{]}$ ) and  $R \ln f_H$  ( $= 33.2 \text{ [J·K}^{-1}\text{·mol}^{-1}\text{]}$ ) in the moderate solute concentration range were approximately the same being around  $18 \text{ [J·K}^{-1}\text{·mol}^{-1}\text{]}$ . It is also noticed that  $Z$  estimated for the very dilute  $TiH_x$  was 27 which was comparable to  $Z = 30$  estimated for the very dilute  $TiD_x$ .

As such, parameter values estimated for  $TiH_x$  and  $TiD_x$  in both the very dilute and the moderate solution concentration ranges look generally consistent among themselves. In contrast, such consistency among the estimated parameters for  $ZrH_x$  and  $ZrD_x$  were not found in Table 5. Although the lacking inconsistency among the estimated statistical

thermodynamic parameter values for  $ZrH_x$  and that for  $ZrD_x$  might be genuine origin and might be interpreted rationally but we cannot rule out the possibility that the results for  $ZrH_x$  and for  $ZrD_x$  were affected by inherent presence of Hf (hafnium) impurity in industrially pure Zr.

### 3.2. Attempts to Estimate $R \ln Zf_x(T)$ Values Taking into Account $T$ -dependence of $f_x(T)$

As represented by Eq. (4), expression for  $\ln f_x(T)$  is a function of  $T$

$$\ln f_x(T) = -\int_0^\infty g(v) \ln\{1 - \exp[-hv/(kT)]\}dv + \ln \rho v_0. \quad (4)$$

As pointed out above, there are scarce information on  $v$  values for atoms  $X$  in non-stoichiometric interstitial solid solution  $MX_x$ . Anyway, using a few available information for values of  $v_H$  [55-57], very crude estimation was attempted to evaluate  $R \ln Zf_H(T)$  values as a function of  $T$  by taking  $v_H = 750 \text{ cm}^{-1}$  assuming the multiplicity of this vibrational mode of H being 3. The estimation results are summarized in Table 6.

**Table 6.** Estimated values for  $R \ln Zf_H(T)$  taking into account  $T$ -dependence of  $f_H(T)$  for some  $MH_x$  compared with estimated values for  $R \ln Zf_H$  according to standard statistical thermodynamic analysis procedure

$MH_x$	$R \ln Zf_H(T)$ [ $J \cdot K^{-1} \cdot mol^{-1}$ ]	$R \ln Zf_H$ [ $J \cdot K^{-1} \cdot mol^{-1}$ ]	Ref.
$LaH_{2+\delta}$	38.1 (450°C) - 35.2 (250°C)	45.4	6
$CeH_{2+\sigma}$	50.7 (600°C) - 46.4 (300°C)	69.1	6
$PrH_{2+\delta}$	51.9 (450°C) - 49.0 (250°C)	59.1	6
$NdH_{2+\delta}$	49.5 (450°C) - 46.6 (250°C)	57.0	6
$SmH_{2+\delta}$	46.9 (450°C) - 44.0 (250°C)	54.3	6
$NpH_{2+\delta}$	37.1 (550°C) - 34.2 (350°C)	45.9	6
$PuH_{2+\delta}$	35.4 (500°C) - 32.5 (300°C)	43.3	6
$AmH_{2+\delta}$	45.3 (600°C) - 41.0 (300°C)	53.5	6
$VH_x$	61.6 (554.0°C) - 59.5 (245.6°C)	64.7	8
$NbH_x$	57.1 (671.0°C) - 54.6 (352.5°C)	61.8	8

In Table 6, it is noticed that  $R \ln Zf_H(T)$  range for any  $MH_x$  system were lower than  $R \ln Zf_H$  value estimated from standard statistical thermodynamic analysis procedure under implicit assumption of  $T$ -independence of  $f_H$ . There is, for the moment, no rational explanation for this.

## 4. Concluding Remarks

Characteristic features of entropy terms appearing in statistical thermodynamic analysis formula for non-stoichiometric interstitial compound  $MX_x$  are reviewed.

On undertaking the statistical thermodynamic analysis for  $MX_x$ , essential first step is to compose plausible partition function  $PF(MX_x)$  to  $MX_x$  by taking realistic distribution model for interstitial non-metallic constituent  $X$  in  $MX_x$  lattice. For this criterion to be fulfilled, it was proposed in the first publication [3] of this series of work to choose the number  $\theta$  of available interstitial sites per  $M$  atom that appears in fundamental formula Eq. (1) in a way to yield

constant  $E(X-X)$  at arbitrary  $T$  over entire homogeneity composition range of  $MX_x$ . The system analyzed in [3] was hypo-stoichiometric nitride of chromium,  $CrN_{0.50-\delta}$ , and  $\theta$  to fulfil the *a priori* assumption of constant  $E(N-N)$  was determined to be 0.5 and the analysis results looked realistic holding rational compatibility with the available thermodynamic parameter values for  $Cr_2N$  phase.

This choice of  $\theta$  parameter value for analysis of  $CrN_{0.50-\delta}$  would mean that available sites for N atom occupation in hcp  $CrN_{0.50-\delta}$  lattice were O-sites and occupation of one O-site by N atom would block occupation by a neighbouring O-site by another N atom (i.e.,  $Z = 2$ ).

Geometrically available number  $\theta_0$  of available interstitial sites per M atom in close-packed  $MX_x$  lattice (*fcc* or *hcp*) would be 1 if the O-sites are occupied or 2 if T-sites are occupied

$$\theta_0(\text{fcc/hcp}; O) = 1, \quad (20)$$

$$\theta_0(\text{fcc/hcp}; T) = 2. \quad (21)$$

On the other hand, that for rather open bcc lattice might be represented by

$$\theta_0(\text{bcc}; O) = 3, \quad (22)$$

$$\theta_0(\text{bcc}; T) = 6. \quad (23)$$

Deviation of  $\theta$  from  $\theta_0$  might be quite significant for some  $MX_x$  as experienced for analysis of primary solid solution  $MH_x$  with bcc lattice structure for Va-group transition metals ( $M = V, Nb, Ta$ ) [5];  $\theta = 0.55$  was chosen for V and Ta (that is  $Z \approx 5$ ) while  $\theta = 0.75$  was chosen (that is,  $Z = 4$ ) for  $M = Nb$  while  $\theta_0(\text{bcc}; O) = 3$ .

Without accepting the proposed *a priori* assumption of constant  $E(X-X)$  at arbitrary  $T$  within a given phase, conclusions drawn from statistical thermodynamic analysis would become with great arbitrariness and meaningless. For example, when  $\theta = \theta_0(\text{hcp}; O) = 1$  (that is, random distribution of N atoms over geometrically available O-sites in hcp lattice of  $CrN_{0.50-\delta}$ ) is accepted for analysis of  $CrN_{0.50-\delta}$ , variation of  $E(N-N)$  with composition of  $x$  at arbitrary  $T$  is derived showing trend of increasing repulsive nearest neighbour N-N interaction with increasing  $x$ . Such situation is not realistic because if  $E(N-N)$  interaction changed with composition  $x$ , phase change should be induced at certain threshold  $x$ .

As such, choice of realistic  $\theta$  representing configurational entropy term of  $MX_x$  is the essential first step of initiating statistical thermodynamic analysis for  $MX_x$ .

Then, as the consequence of the statistical thermodynamic analysis carried out on the basis of this simplifying *a priori* assumption, apparently realistic conclusion was drawn for value of the term  $R \ln Zf_H$  for  $MH_x$  of Va-group metals in spite of different  $\theta$  values assigned for  $M = V$  and  $Ta$  and for  $M = Nb$  (cf. Table 1) that fell at around  $63.5 \pm 2 \text{ [J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}]$ . As also seen in Table 1,  $Q$  values evaluated for these  $MH_x$  were also comparable to each other being around  $-227 \pm 3 \text{ [kJ} \cdot \text{mol}^{-1}]$ .

Discussion on contribution of electronic term  $R \ln Zf_x$  of tightly bound electrons in  $MX_x$  lattice remains still rather

primitive and imperfect. It is hoped that this aspect is exploited further being assisted by compiled data on vibrational frequency  $\nu$  of interstitial constituent X in  $\text{MX}_x$  lattice.

As reviewed in this paper, entropy terms in multi-component alloys including non-stoichiometric interstitial alloys are consisted of contribution from tightly bound electrons besides contribution from configuration of constituent atoms over crystal lattice points.

Unlike enthalpy term, entropy term is not straightforwardly measurable by experiment for alloys. For example, in spite of strategic importance of entropy term in characterizing fashionable high-entropy alloys, report appears to be scarce on values of entropy terms in published works on high-entropy alloys.

The concept of high-entropy alloy was born in 2004 by Cantor et al. [58] being defined originally as “equi-atomic multi-component alloy”. After being christened with distinctive name “high-entropy alloys” [59], research on this category of alloys became highly fashionable. Nevertheless, in available works published on high-entropy alloys [60], explicit quantitative evaluation for entropy term did not seem to be made very seriously. This is somewhat peculiar and surprising.

High-entropy alloys are classified in substitutional alloys and thence the analysis procedure used in this work for interstitial alloy systems cannot be applied straightforwardly. However, if effort is invested to evaluate somehow contribution from electronic terms as well as from atomic configuration term, rate of progress in high-entropy alloy might be raised.

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## Appendix / Symbols

### <universal constants>

$R$ : universal gas constant ( $= 8.31451 \text{ [J mol}^{-1} \text{ K}^{-1}]$ ),

$h$ : Planck constant ( $= 6.6260755 \times 10^{-34} \text{ [J s]}$ ),  
 $k$ : Boltzmann constant ( $= 1.380658 \times 10^{-23} \text{ [J K}^{-1}]$ ),

### <materials constants>

$m_X$ : mass of X atom,  
 $\rho$ : nuclear spin weight,  
 $\Theta_r$ : characteristic temperature for rotation of  $X_2$ ,  
 $\Theta_v$ : characteristic temperature for vibration of  $X_2$ ,  
 $\nu_0^*$ : electronic state of normal state of  $X_2$  molecule,  
 $D(X_2)$ : dissociation energy of  $X_2$  molecule per mole,  
 $\beta$ : factor determined from crystal structure consideration,  
 $\theta_0$ : geometrically available number of interstitial site per M in  $\text{MX}_x$ ,  
 $\nu_0$ : statistical weight of tightly bound electrons around X in  $\text{MX}_x$ ,  
 $\nu$ : vibrational frequency of X atom in  $\text{MX}_x$  lattice,  
 $g(\nu)$ : distribution function,

### <experimentally measurable macroscopic parameters>

$p(X_2)$ : equilibrium pressure of ideal gas  $X_2$ ,  
 $T$ : absolute temperature [K],  
 $x$ : composition (X/M atom ratio  $= n_X/n_M$ ) in  $\text{MX}_x$ ,  
 $n_X$ : number of X atoms in  $\text{MX}_x$  lattice,  
 $n_M$ : number of M atoms in  $\text{MX}_x$  lattice,

### <atomistic parameters to be evaluated>

$Q$ : degree of stabilisation of X atom in  $\text{MX}_x$  lattice with reference to isolated X and M atoms in vacuum,  
 $E(i-j)$ : interaction energy between  $i$  and  $j$  atoms in  $\text{MX}_x$  lattice,  
 $E(\text{MX}_x)$ : lattice energy,  
 $f_X(T)$ : partition function of X atom in  $\text{MX}_x$  lattice,  
 $f_M(T)$ : partition function of M atom in  $\text{MX}_x$  lattice,  
 $K$  &  $g$ : parameters determined by Equations, (1) & (2), from the experimental  $PTC$  data for an assigned value of  $\theta$ ,

### <a factor to be assigned a priori>

$\theta$ : number of the interstitial sites per M atom available for occupation by X atoms in  $\text{MX}_x$  lattice,

### <a resultant model parameter referring to extent of blocking of interstitial sites>

$Z$ : extent of blocking of interstitial sites by X in  $\text{MX}_x$  lattice ( $= \theta_0/\theta$ ); that is, when one interstitial site in  $\text{MX}_x$  is occupied by an X atom,  $(Z - 1)$  neighbouring interstitial sites are blocked from occupation by other X atoms.

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