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Parameter identification and uncertainty evaluation in quasi-dynamic test of solar thermal collectors with Monte Carlo method

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A B S T R A C T

This work presents a comparison between two methods used for parameter identification and calculation of parameter uncertainty applied to the measured data of solar thermal collectors when tested according to ISO 9806:2017. One method is using a weighted least square (WLS) fit and the partial derivative approach described in the GUM (the Guide to the Expression of Uncertainty in Measurement). The second is using the Monte Carlo (MC) method, also described in GUM. Uncertainty evaluation by Monte Carlo method is based on a probabilistic approach and is an alternative way for identification of parameters and determination of the uncertainties. In this work the results were obtained according to Quasi-Dynamic Test (QDT) method for a flat plate collector and an evacuated tubular collector. The least squares (LS) method and a nonlinear regression method (*MPFit*) are used in the identification of parameters for each iteration of the MC method. For the implemented MC method computation times are also discussed. One disadvantage of the MC method is the computation time which depends on the number of samples in the experimental test quantity files, however with this study we think that the advantages of the MC method outweigh the disadvantages, and it is useful even as a complementary tool in QDT testing of collectors.

Nomenclature and abbreviations

A_G	gross area of collector	m^2
a_1	heat loss coefficient at $\vartheta_m - \vartheta_a = 0$	W/(m^2·K)
a_2	temperature dependence of the heat loss coefficient	W/(m^2·K^2)
a_3	wind speed dependence of the heat loss coefficient	J/(m^3·K)
a_4	sky temperature dependence of the heat loss coefficient	
a_5	effective thermal capacity	J/(m^2·K)
a_6	wind dependence in the zero loss efficiency	s/m
a_7	wind speed dependence of IR radiation exchange	W/(m^2·K^4)
a_8	radiation losses	W/(m^2·K^4)
b_0	constant for the calculation of the incident angle modifier	–
E_L	long wave irradiance (λ > 3 μm)	W/m^2
G_b	direct solar irradiance (beam irradiance)	W/m^2
G_d	diffuse solar irradiance	W/m^2
$K_b(\theta)$	incidence angle modifier for direct radiation	–
$K_b(\theta_L, \theta_T)$	incidence angle modifier for direct radiation	–
K_d	incidence angle modifier for diffuse radiation	–
$\dot{m}$	mass flow rate of heat transfer fluid	kg/s

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(*continued*)

N	constant for the calculation of the incident angle modifier	–
$\dot{Q}$	useful power extracted from collector	W
$\dot{Q}/A_G$	specific useful energy extracted from the collector	W/m^2
T	absolute temperature	K
u	surrounding air speed	m/s
$\eta_{0,b}$	peak collector efficiency (η_b at $T^*m = 0$), reference to T^*m based on beam irradiance G_b	
σ	Stefan-Boltzmann constant	W/(m^2·K^4)
θ	angle of incidence	degrees
θ_L	longitudinal angle of incidence	degrees
θ_T	transversal angle of incidence	degrees
γ	collector azimuth angle (0 = south, east negative)	degrees
ϑ_m	mean temperature of heat transfer fluid	°C
ϑ_{in}	collector inlet temperature	°C
ϑ_a	ambient or surrounding air temperature	°C
X^2	X-square	
x_i	independent variable	
y_i	dependent variable	
m_i	Parameters	
σ_i	standard deviation	
$u_c(y)$	composite standard uncertainty	

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$u(x_i)$	standard Type A or B uncertainty
IAM	Incidence angle modifier
LS	Least square
QDT	Quasi-Dynamic Test
WLS	Weighted least squares
MC	Monte Carlo.
MC_{LS}	MC method with the LS fit
MC_{MPFit}	MC method with MPFit
MPFit	Library routines implementing a nonlinear regression method

1. Introduction

In the design of solar thermal systems, the characterization of the thermal performance of the components of the system, namely the solar thermal collector, is crucial. For this characterization, testing of the collector is necessary. Since the 70′s testing methods have been developed and these methods have been published in regional [1,2] and international standards. ISO 9806, with an initial publication in 1994 [3], has been revised and the most recent version dates from 2017 [4]. The thermal performance test is now applicable to fluid heating solar collectors which are clearly stated in the scope of the standard, namely, air heating solar collectors and liquid heating collector, including hybrid solar collectors co-generating heat and electric power,[1] as well as to solar collectors using external power sources for normal operation and/or safety purposes, but it does not cover evaluation of electric power generation. The ISO 9806 considers two test methodologies, assumed as equivalent, Steady State Test Method and Quasi-Dynamic Test Method [5–8].

The standard ISO 9806:2017 describes with detail the testing installation (section 22) and mounting of the collector for testing (section 20), instrumentation, i.e., measurement equipment requirements (section 21), testing conditions and the thermal performance test procedures (section 23). Additionally, the standard includes information on the computation of collector parameters describing the heat balance equations that apply to both steady state test method (SST) and quasi-dynamic test method (QDT) (section 24) making clear which parameters need to be identified. It refers that these parameters "shall be deduced by statistical least square fitting of the measured data points".

For determination of the uncertainty of the parameters the standard refers to its Annex D where it introduces a proposed weighted least square (WLS) fit. This method was already applied to the QDT by Ref. [8] considering the requirements of EN 12975–2:2006. One of the advantages of this method in relation to simple LS method is that WLS allows to include the influence of uncertainties for each point in the evaluation of parameters. However, to do that is necessary to calculate the first order derivatives to evaluate the respective uncertainties.

The present work has the objective to show and evaluate the use of Monte Carlo (MC) method as an alternative way for identification of parameters and determination of the uncertainties. One advantage of using the MC method over WLS is that it allows to evaluate the parameters as well as its uncertainties. The MC method performs random sampling from probability distribution of the input quantities and therefore it is not necessary to calculate the first order derivatives for calculation of uncertainties. In addition, it provides a probability density function for the output quantity as the result, from which the coverage interval can be determined.

This work is focused on liquid heating collectors and will present results of tests according to QDT method for a flat plate collector and an evacuated tubular collector discussing aspects related to the use of the WLS fit for parameter identification and calculation of parameter uncertainty and comparing with the use of the Monte Carlo method [9]. The identification of parameters for each iteration of the MC method was developed to use the LS fit or the nonlinear regression method, included in the *MPFit* library routines [10]. To the best of our knowledge, there are no previous works that explore the use of the MC method in identifying parameters and evaluating uncertainties for solar thermal collectors tested using the quasi-dymanic test method according to ISO 9806:2017. In Ref. [11] the MC method is used to estimate the uncertainty of the expected annual energy output of a solar thermal system when using the test method according to ISO 9459–5:2007 [12].

This paper is organized as follows: Section 2 introduces the quasi-dynamic test method for the characterization of the solar collector's thermal performance accordingly the standard ISO 9806:2017; Section 3.1 introduces the parameter identification and uncertainty assessment using the WLS and Section 3.2 describes the MC methods in general terms; Section 4.1 presents the specific implementation of MC method in the case of the parameter identification for solar thermal collectors and Section 4.2 presents the results achieved for a flat plate collector and for an evacuated tubular collector, comparing these results with the ones achieved with the WLS, LS and nonlinear regression methods, discussing some of the key points in using the MC method. Section 5 concludes the paper.

2. Solar collector thermal performance

The standard ISO 9806:2017 considers two types of test methods for the characterization of the solar collector's thermal performance, the steady state test (SST) method and quasi-dynamic test (QDT) method.

Steady-state test method was the first developed and it was the test method considered in the first version of ISO 9806 [3] and, as well in ASHRAE 93:2003 [2], and other national standards.

Quasi-Dynamic test method was developed later [13,14] and was first considered on the standard EN 12975–2:2006 [1]. Later introduced on ISO 9806:2013 superseding the European Standard.

The QDT method allowed a better characterization of concentrating collectors since it allows the consideration of the two solar irradiance components, direct and diffuse irradiance.

When compared qualitatively, the QDT has some advantages over the stationary test method. The conditions required for the QDT are like the real operating conditions of a collector, since test days with diffuse radiation are included. On the other hand, given that the test conditions are less restrictive, the QDT can be applied to a greater number of collectors within the same period of time [8], while allowing an accurate and complete characterization of the collector's performance.

The two test methods have been studied and compared [5,8] and are considered equivalent. The ISO 9806:2017 reformulates the heat balance equations for both methods making them formally equivalent if normal incidence of beam irradiance is considered.

This work focuses on the QDT method and the heat balance equations for the method are presented in this section.

2.1. Quasi-Dynamic Test (QDT) method

The heat balance equation, according to equation (13) of ISO 9807:2017, for the QDT method considers the two components of solar irradiance, direct and diffuse irradiance, with the corresponding incidence angle modifiers. Thus, the two first right terms of the heat balance equation are $\eta_{0,b}K_b(\theta_L,\theta_T)\ G_b + \eta_{0,b}K_dG_d$. For collectors with concentration ratio lower than 20 the mandatory parameters are $\eta_{0,b}, K_b(\theta_L,\theta_T)$, K_d, a_1, a_2 *and* a_5 and the equation to consider is:

$$\frac{\dot{Q}}{A_G} = \eta_{0,b}K_b(\theta_L,\theta_T)\ G_b + \eta_{0,b}K_dG_d - a_1(\vartheta_m - \vartheta_a) - a_2(\vartheta_m - \vartheta_a)^2 - a_5\frac{d\vartheta_m}{dt} \quad (1)$$

In this case both IAM and collector thermal capacitance are determined simultaneously to the other parameters characterizing the collector thermal performance.

[1] Usually referred as PVT collectors.

The Incidence Angle Modifier (IAM) is defined as the ratio of the peak efficiency at a given angle of incidence and the peak efficiency usually at normal incidence, and for the QDT method two components are considered, i.e., the beam incidence angle modifier:

$$K_b(\theta_L, \theta_T) = \frac{\eta_{0,b}(\theta_L, \theta_T)}{\eta_{0,b}(\theta_L = 0, \theta_T = 0)} \tag{2}$$

The diffuse incidence angle modifier, K_d, for diffuse irradiance is considered isotropic.

For collectors with bi-axial geometry, it is necessary to measure the effects of the incident angle in more than one direction to adequately characterize IAM, which can be estimated by the product of two incidence angle modifiers, K_T and K_L for the two perpendicular symmetrical planes (transversal and longitudinal, respectively) according to Ref. [15]:

$$K_b(\theta_T, \theta_L) = K_b(\theta_T, 0) \times K_b(0, \theta_L) \tag{3}$$

ISO 9806:2017 allows the identification of IAM as a list of individual values, tabulated in steps of 10°, but also with analytical expressions. In this work the longitudinal component will be considered described by an expression proposed in Ref. [16]:

$$K_b(\theta_L) = \left[1 - b_0\left(\frac{1}{\cos\theta_L} - 1\right)\right] \tag{4}$$

where $b_0 > 0$ is the incidence angle modifier coefficient and is adjusted to experimental data. This expression is used to expand the 1st term of equation (1):

$$\frac{\dot{Q}}{A_G} = \eta_{0,b}G_b - \eta_{0,b}b_0\left(\frac{1}{\cos\theta_L} - 1\right)\sum_{i=1}^{n} K_{\theta T_i}G_{b_i} + \eta_{0,b}K_dG_d - a_1(\vartheta_m - \vartheta_a) - a_2(\vartheta_m - \vartheta_a)^2 - a_5\frac{d\vartheta_m}{dt} \tag{5}$$

One of the approaches for modelling IAM for bi-axial collectors is to use equation (5) considering that $K_b(0,\theta_L)$ is given by equation (4) and factorizing $K_b(\theta_T,0)$.

equation (3) will be given by:

$$K_b(0, \theta_L) \times K_b(\theta_T, 0) = K_b(\theta_L) + K_b(\theta_L)K_b(5^o \le \theta_T < 10^o) + K_b(\theta_L)K_b(10^o \le \theta_T < 15^o) + \ldots \tag{6}$$

This approach is based on the use of “dummy variables” in classical multilinear regression [15] which was introduced for the QDT method by Ref. [14] and is the methodology used here in the treatment of data from tests on bi-axial collectors by the quasi-dynamic method.

Other models for the IAM can also be used [17,18]. The ISO 9806:2017 standard suggests the nonlinear model given by equation (7) which involve using a non-linear regression method.

$$K_b(\theta) = 1 - tan^N\left(\theta/2\right) \tag{7}$$

3. Parameter identification and uncertainty assessment

3.1. Weighted Least Squares method

For determination of collector parameters, a multilinear regression method can be applied to equation (1) or (5).

These equations have a single dependent variable $\frac{\dot{Q}}{A_G}$ and multiple independent variables. A description of Least Squares (LS) Method can be found in Ref. [19] and a description of Weighted Least Squares (WLS) method can be found in Ref. [20]. The application of the WLS method for the SST has been shown in Ref. [21–23]. The application of WLS method in the QDT method was also shown by Ref. [8,24]. Advantages of application of WLS method can be found in Ref. [24]. ISO 9806:2017, includes a description of this method in the informative Annex D.

In WLS method the values of the parameters are estimated from the minimization of the quadratic sum of the residuals:

$$X^2 = \sum_{i=1}^{n}\left[\frac{[y(i) - (m_1x_1(i) + m_2x_2(i) + \cdots + m_nx_n(i)]^2}{\sigma_i}\right] \tag{8}$$

One of the advantages of this method in relation to simple LS method is that this method allows not only quantitative but also qualitative fit since a weighting (σ_i) is attributed to the influence of each point in the parameter estimates. This allows to include measurement uncertainties in the evaluation of parameters and the respective uncertainties.

In cases where the experimental data are subject to measurement uncertainties not only in y_i but also in x_i, σ_i represents the uncertainty in the form of standard uncertainty, for each point i given by

$$\sigma_i{}^2 = \sigma_{y_i}^2 + \sum_{j=1}^{n} m_j\sigma_{x_{j_i}}^2 \tag{9}$$

where σ_{yi} and σ_{xi} are respectively the standard uncertainties of y and x for each point i.

To determine the uncertainties resulting from the measurement process the orientations of GUM (Guide to the expression of Uncertainty in Measurement [25], were followed as well as Annex D of ISO 9806.2017.

The ISO 9806: 2017 standard establishes a maximum standard uncertainty for measurements. Table 1 shows the maximum standard uncertainty limits defined by the standard.

The expression for the uncertainty that characterizes the experimental power per unit area of the collector in equation (1) is given by:

$$(\dot{Q}/A_G)_i = \frac{\dot{m}_iC_p(\vartheta_{out} - \vartheta_{in})_i}{A_G} \Rightarrow$$
$$u(\dot{Q}/A_G)_i = \sqrt{\left(\frac{\partial(\dot{Q}/A_G)}{\partial\dot{m}}u(\dot{m}_i)\right)^2 + \left(\frac{\partial(\dot{Q}/A_G)_i}{\partial(\vartheta_{out} - \vartheta_{in})_i}u(\vartheta_{out} - \vartheta_{in})_i\right)^2 + \left(\frac{\partial(\dot{Q}/A_G)_i}{\partial A_G}u(A_G)\right)^2} \tag{10}$$

Table 1
– Maximum standard uncertainty of measurement according to ISO 9806:2017.

	Maximum standard uncertainty
A_G [m^2]	0.3 %
G [W/m^2]	2.45 %
$\dot{m}$ [kg/s]	1 %
ϑ_{in} [K]	0.1
ϑ_{out} [K]	0.1
T_a [K]	0.5
ΔT [K]	0.05
v [m/s]	0.5
Δt [s]	0.2 %

The results of the uncertainties thus calculated and applied in equation (8) represent the standard uncertainties for each point i and are used in the calculation of the parameters by the method of WLS.

3.2. Monte Carlo (MC) method

The Monte Carlo (MC) method is a statistical method based on the propagation of probability distribution functions (PDF) through the repetition of successive simulations (random samples), a high number of times, to find solutions to a given problem. This method is commonly used to obtain numerical approximations of complex functions in which it is difficult, or even impossible, to obtain an analytical or deterministic solution [11]. It can also be used where the objective is to determine a relational model between dependent and independent variables. In our case and since the model already exists, the MC method is used instead as a tool for achieving the uncertainties of the model parameters.

This methodology is based on the GUM supplement JCGM 101:2008 [26] which proposes the propagation of probability distributions of measurements as a basis for the assessment of uncertainties. This treatment is applied to a mathematical model with a determined number of input quantities (measured values) and a single output.

The number of Monte Carlo iterations (M) correspond to the number of evaluations performed on the model. M can be chosen a priori or by an adaptive method. For example, it is known that with a value of $M = 10^6$ it is expected to deliver an interval with a 95 % confidence level [26].

The principle of determining uncertainty using the MC method involves the assignment of a probability density function (PDF) to the independent random variable X_i, based on the information taken from the various observations (evaluation of Type A) and/or based on judgment using other information, such as reference values or calibrations (evaluation of Type B uncertainty). M vectors $X_{r,i}$, $r = 1, \ldots, M$ are generated from the probability density functions where each vector $X_{r,i}$ contains the n estimate of PDF for the input quantities X_i. The propagation of the PDF for the different $X_{r,i}$ must be made through the model

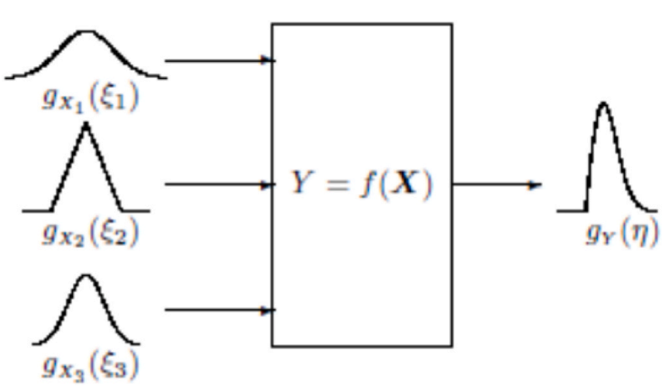

Fig. 1. – Graphical exemplification of Monte Carlo method (adapted from Ref. [26]).

(propagation by the model) to obtain the PDF of Y_r. The model values will then come:

$$Y_r = f\left(X_{r,i}\right) \; r = 1, \ldots, M; \forall r : i = 1, \ldots, n \tag{11}$$

The estimate of y by the MC method is obtained by averaging:

$$\overline{y} = \frac{1}{M}\sum_{r=1}^{M} Y_r \tag{12}$$

and the standard deviation associated with y is obtained by the expression:

$$u(\overline{y}) = \sqrt{\frac{1}{M-1}\sum_{r=1}^{M}\left(Y_r - \overline{y}\right)^2} \tag{13}$$

A schematic of the application of the Monte Carlo method is shown in Fig. 1.

The calculation process by MC method can be summarized by the following procedure.

1. Select the number of MC iterations to be performed. It is recommended to use M = 10^6 to estimate a 95 % coverage interval for the output quantity such that this length is correct to one or two significant decimal digits [26].
2. Generate M vectors of the n input quantities X_i (X_1, X_2, …, X_n), by sampling the PDF assigned to each input parameter.
3. For each of these vectors, form the corresponding Y model, producing M model values (Y_1, Y_2, …, Y_M).
4. Classify these M model values in non-decreasing order, using the values obtained to generate G (the PDF of the dependent variable y).
5. Use G to form a y estimate of Y, and the standard uncertainty $u(y)$ associated with y.
6. Use G to form a confidence interval suitable for Y, for a stipulated probability of confidence. This last step can be optional, if the value chosen for M has already associated a certain degree of confidence.

In the present case the independent variables X_i are ϑ_a, ϑ_i, ϑ_o, $\dot{m}$, G_b, G_d, …according to equation (1). This means that in step 2 we will have matrixes M x n where n is the number of measurements resulting from performing the test sequences according to ISO 9806:2017.

For each r, the parameters of the heat balance equation (1) considered are determined by the LS or *MPFIT* method and a set of M values are available for each set of parameters ($\eta_{0,b}$, $K(\theta)$, K_d, a_1, a_2, a_3, a_4, a_5, a_6, a_7 *and* a_8). Applying equations (12) and (13) it is possible to obtain the values of the parameters and their corresponding uncertainty.

Also, for each parameter it is possible to determine its PDF by applying step 4 to each parameter.

4. Results and discussion

4.1. Implementation of MC method

The implemented MC method can be described by the following

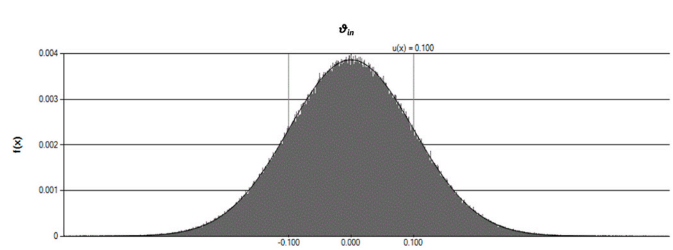

Fig. 2. The generated probability values with a normal distribution to be assigned to the test quantity ϑ_{in} with 10^6 iterations of the MC method.

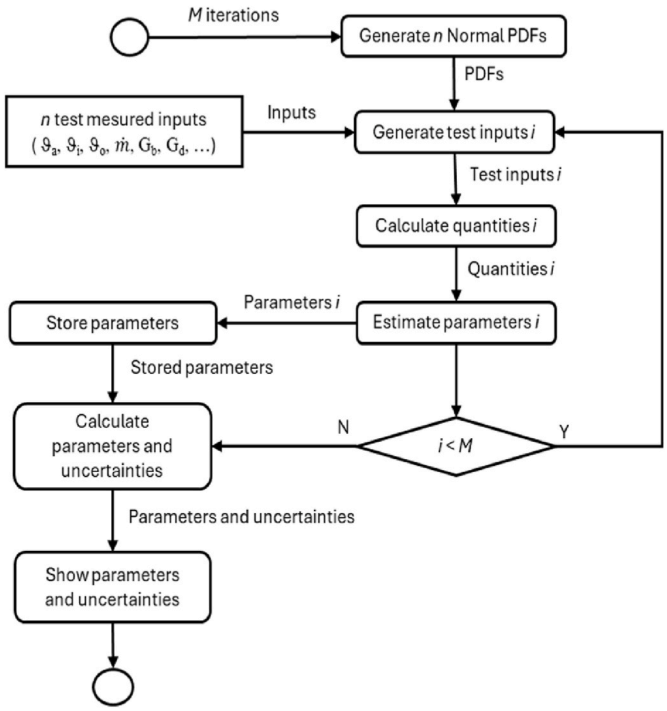

Fig. 3. A data flow diagram of the MC method.

steps.

1. Assignment of a symmetric normal distribution to each measured quantity. The mean for the PDF is the experimental test quantity value (x_i), while its standard deviation value (u_i) is given in Table 1. As already referred in section 3.1, the ISO 9806:2017 standard establishes a maximum uncertainty for measurements. The uncertainty to be considered is obtained by assigning a symmetrical normal distribution to the independent quantities (which covers type A and type B uncertainties).
2. Given all the new calculated experimental quantities, a propagation of probability distribution is performed (propagation by the model) and new quantity values are calculated. For instance for the experimental quantity (ϑ_m- ϑ_a)$_{i,r}$ comes:

$$(\vartheta_m - \vartheta_a)_{i,r} = \frac{\vartheta_{ini,r} + \vartheta_{outi,r}}{2} - \vartheta_{ai,r} \quad (14)$$

Where $\vartheta_{ini,r}$, $\vartheta_{outi,r}$ and $\vartheta_{ai,r}$ are the new calculated experimental quantities for the *r* iteration of the MC method. Note that these calculations are performed for all quantities to be used in the linear regression, including a new value for the collector power.

3. Identification of new parameters by linear regression using the LS (Least Square) method.

The steps described above are repeated *M* times where *M* is the number of iterations for the MC method. Finally, the *M* parameters obtained are averaged using equation (12) and the uncertainties calculated through equation (13). Fig. 3, shows a data flow diagram of the implemented MC method.

The MC method was developed with a sequential version and with a parallel version to reduce the execution times of the algorithm. The parallel version was implemented using the Task Parallel Library (TPL) available in the.NET Framework 4 [27], using a static partitioning schema to avoid synchronization between tasks. In this schema the *M* iterations are equally divided into one range per task. The number of tasks (as well as the number of iterations *M*) to be used can be set by the user before each computation of the MC method. With the parallel version, the execution times are about 62–68 % lower than with the sequential version. Calculations were made in a computer with an Intel® Core™ i7-8700 3.20 GHz processor with 16 Gb of RAM. All the computations were done using 6 tasks which equals the number of physical cores of the CPU. We have also registered the execution times for a different number of tasks, and we didn't find an advantage in increasing the number of tasks above 6. All results presented in this work were calculated with the parallel version of MC method. The same results can be achieved with the sequential version using the same generated PDF.

4.2. Influence of number of iterations of Monte Carlo method in QDT results

4.2.1. Quasi-dynamic test results of a flat plate collector with the MC method

Table 2 presents the mandatory parameters (and their uncertainties), as a function of the number of iterations, calculated with the MC method with the LS fit (MC$_{LS}$). The table also shows the results achieved with the LS and WLS to compare with.

Parameter values vary little with the number of iterations and are closer to the values obtained with the LS method, not too different compared with the values achieved with the WLS method. The major differences occur for parameters b_0 and a_5 when comparing with the values achieved with the WLS method. However, the greater differences are from the uncertainty parameter values, where larger values were achieved with the MC method for the uncertainties $u(\eta_0)$, $u(b_0)$ and $u(K_d)$ and lower values for the uncertainties $u(a_1)$, $u(a_2)$ and $u(a_5)$. Parameters a_1 and a_2 are linked to the temperature difference given by equation (14) and its squared values, respectively. The resulting PDF, for parameter a_1, by applying equation (14) has a normal distribution, with an uncertainty around 0.505 °C, which equals the theoretical computed

Table 2
Mandatory parameters (and their uncertainties) calculated with the MC$_{LS}$ method for flat plate solar collector.

Iterations:	10^3	10^4	10^5	10^6	10^7	WLS	LS
η_0	0.679	0.680	0.679	0.680	0.680	0.678	0.679
$u(\eta_0)$	0.020	0.021	0.020	0.020	0.020	0.001	0.001
b_0	0.092	0.093	0.092	0.092	0.092	0.100	0.092
$u(b_0)$	0.033	0.033	0.033	0.033	0.033	0.005	0.006
K_d:	0.81	0.81	0.81	0.81	0.81	0.83	0.81
$u(K_d)$	0.033	0.033	0.033	0.033	0.033	0.009	0.010
a_1	2.77	2.77	2.77	2.77	2.77	2.80	2.77
$u(a_1)$	0.04	0.04	0.04	0.04	0.04	0.05	0.05
a_2	0.019	0.019	0.019	0.019	0.019	0.018	0.019
$u(a_2)$	0.0003	0.0003	0.0003	0.0003	0.0003	0.0006	0.0006
a_5	13810	13810	13811	13812	13812	13880	13807
$u(a_5)$	216	220	220	219	219	398	450
$u(Q/A_G)$	9.498	9.499	9.499	9.499	9.499	1.466	9.456
Time (s):	6.68	28.92	286.66	2949.37	30001.19	0.015	0.015

Table 3
Mandatory parameters (and their uncertainties) calculated for an averaging interval of 30 s according to the standard ISO 9806:2017.

Iterations:	10^3	10^4	10^5	10^6	WLS	LS
η_0	0.678	0.678	0.678	0.678	0.673	0.673
$u(\eta_0)$	0.020	0.020	0.020	0.020	0.001	0.001
b_0	0.095	0.095	0.096	0.096	0.095	0.095
$u(b_0)$	0.034	0.034	0.034	0.034	0.005	0.005
K_d	0.85	0.85	0.85	0.85	0.85	0.85
$u(K_d)$	0.035	0.035	0.034	0.034	0.009	0.009
a_1	2.67	2.67	2.67	2.67	2.70	2.70
$u(a_1)$	0.04	0.04	0.04	0.04	0.04	0.04
a_2	0.020	0.020	0.020	0.020	0.019	0.019
$u(a_2)$	0.0003	0.0003	0.0003	0.0003	0.0006	0.0006
a_5	10506	10505	10501	10503	10421	10425
$u(a_5)$	220	218	219	219	144	144
$u(Q/A_G)$:	25.889	25.888	25.883	25.886	1.012	25.669
Time (s):	56.65	576.68	5826.72	59988.23	0.171	0.058

value, given by equation (15) and applying the values given in Table 1. The resulting PDF of the squared values of the temperature difference has a normal distribution for mean values above 10 °C and approaches a chi-squared distribution, as the mean values decrease below 10 °C.

$$u(\vartheta_m - \vartheta_a) = \sqrt{u^2(\vartheta_m) + u^2(\vartheta_a)} = 0.505 \quad (15)$$

Another big difference is from the uncertainty of the estimated collector power. With the WLS method we obtain a value of 1466 W/m^2 while with the MCM the computed value is around 9,5 W/m^2. This difference is proportional to the value of the weight (σ) assigned by WLS method in computing the parameters.

The differences between LS and MC methods are like the WLS and MC methods for the parameters values as well for the uncertainties, except for the uncertainty of the estimated collector power which is practically the same.

The experimental test quantities used in this work were achieved with averaging intervals of 5 min, according to the previous test standard EN 12975-2: 2006. It is the practice of LNEG to acquire and record all data with a sampling rate of 3 s, which agrees with ISO 9806:2017. An averaging interval of 5 min is equivalent to 100 measurements. Data treatment considering an averaging interval of 30 s was also performed considering the reference to this averaging period given in ISO 9806:2017, section 23.5.3. The results are presented in Table 3. Measured computation times are much longer than for the 300 s average because the number of samples are greater (can be up to 10 times larger), so the 10^7 case was not computed (it would take about 200 h).

Parameter values vary very little with the number of iterations. Interesting to note is the much closer values between the LS and WLS methods, except for $u(Q/A_G)$, for the reason already stated above. Greater differences are from the uncertainty parameter values, where larger values were achieved with the MC_{LS} method, except for the uncertainties $u(a_1)$, $u(a_2)$ and $u(a_5)$, which are closer to the ones achieved with LS and WLS methods.

Table 4, shows the differences in percent for parameters, between averaging interval of 300 s and 30 s. Minus sign indicates a smaller value for the averaging interval of 300 s. For the parameter values, small variations exist among methods. The larger difference is for parameter a_5 which value is about 25 % greater for the 300 s average. In fact, greater differences are obtained for different averaging than those obtained by using different regression methods for the same averaging time. A study about the impact of averaging interval in parameter identification using the WLS method for the QDT, can be found in Ref. [28]. It seems that there is no advantage in using averaging intervals of 30 s. As stated above, one problem of using the averaging interval of 30 s with the MC method is the largger number of samples.

Table 4
Differences in percent for parameters between averaging interval of 300 s and 30 s.

Method	η_0	b_0	K_d	a_1	a_2	a_5
10^6	0.2	−3.7	−4.7	3.4	−7.4	24.0
WLS	0.8	5.3	−2.2	3.6	−5.1	24.9
LS	0.9	−3.4	−4.6	2.3	−1.8	24.5

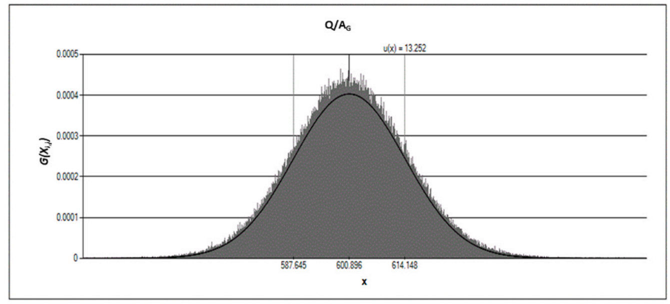

Fig. 4. The estimated probability distribution function obtained for the estimated collector power of around 600 W/m^2, with 10^6 iterations.

4.2.2. Confidence interval

The estimated PDFs for the estimated collector power values are all normal as can be seen in the example shown in Fig. 4. Using equation (11) and the Central Limit Theorem we can assume that $\frac{\dot{Q}}{A_G}$ will also be normal and we can calculate its level of confidence. For a level of confidence of $p = 95\%$ we get a coverage factor (k_p) of 1.96 resulting in an interval for $\frac{\dot{Q}}{A_G}$ and for 10^6 iterations in Table 2, of:

$$\frac{\dot{Q}}{A_G} \pm 1.96 \times u\left(\frac{\dot{Q}}{A_G}\right) \mathrm{W}/m^2 = \frac{\dot{Q}}{A_G} \pm 1.96 \times 9.5 \approx \frac{\dot{Q}}{A_G} \pm 18.62 \ \mathrm{W}/m^2$$

Note that the PDF shown in Fig. 4 has an uncertainty value of 13.252, greater than the uncertainty value around 9.5 W/m^2 used above. The uncertainties given in Table 2 are calculated as the average value of all $u(\dot{Q}/A_G)$ values obtained for the calculated $\dot{Q}/A_G$ in each iteration of the MC_{LS} method while the uncertainty in Fig. 4 correspond to the achieved $(\dot{Q}/A_G)_i$ (see equation (6) or $f(X_{r,i})$ using equation (11) for $i = 32$ and $r = 10^6$ iterations. The average value of $u(\dot{Q}/A_G)$ calculated in this way is around 12.8 W/m^2.

4.2.3. MC method with a nonlinear regression method

Table 5 shows the mandatory parameters (and their uncertainties) calculated with the MC method (MC_{MPFit}) but using the nonlinear regression method [10] included in the *MPFit library* routines available at [29]. Parameter $K_b(\theta)$ was computed using equation (7). The last column presents the results calculated with the function *lscurvefit* of MATLAB®, used for solving nonlinear data-fitting problems. These results are very close to the ones obtained with the *MPFit* function as shown in the table.

Parameter values vary very little with the number of iterations and are very closer to the values obtained with one iteration. With regard to uncertainties, larger values were achieved with the MC method for $u(\eta_0)$, $u(N)$ and $u(K_d)$. Parameter values and their uncertainties are also closer to the values presented in Table 2, except for parameter N that cannot be compared with parameter b_0.

Fig. 5 show the PDFs obtained for parameters b_0 and N that results from applying the normal PDFs referred in section 5.1.1, where Fig. 2 shows an example for ϑ_{in} with 10^6 iterations. For parameter b_0 the PDF is normal while parameter N has a lognormal PDF. This means that we can assign a new value for N, given by mean of its PDF, which gives a value around 3.9 instead of the value of 4.1 presented in Table 2 and for the uncertainty a value of 0.345. For the other parameters the obtained PDFs are all normal.

Table 5
Mandatory parameters (and their uncertainties) calculated with the MC_{MPFit} method.

Iterations:	10^3	10^4	10^5	10^6	10^7	*MPFit*	lscurvefit
η_0	0.677	0.677	0.677	0.677	0.677	0.676	0.676
$u(\eta_0)$	0.020	0.021	0.021	0.021	0.021	0.001	0.001
N	4.119	4.082	4.064	4.060	4.061	3.880	3.880
u(N)	2.214	2.122	1.917	1.873	1.875	0.082	0.082
K_d:	0.82	0.82	0.82	0.82	0.82	0.82	0.82
$u(K_d)$	0.033	0.033	0.033	0.033	0.033	0.009	0.009
a_1	2.77	2.78	2.78	2.78	2.78	2.78	2.78
$u(a_1)$	0.039	0.039	0.038	0.039	0.039	0.048	0.048
a_2	0.019	0.019	0.019	0.019	0.019	0.019	0.019
$u(a_2)$	0.0003	0.0003	0.0003	0.0003	0.0003	0.0006	0.0006
a_5	13934	13927	13928	13928	13928	13935	13934
$u(a_5)$	213.66	215.66	214.20	213.65	213.60	446	446
$u(Q/A_G)$:	9.406	9.401	9.400	9.400	9.400	9.357	9.357
Time (s):	0.644	5.493	59.219	658.629	6774.53	0.002	0.000

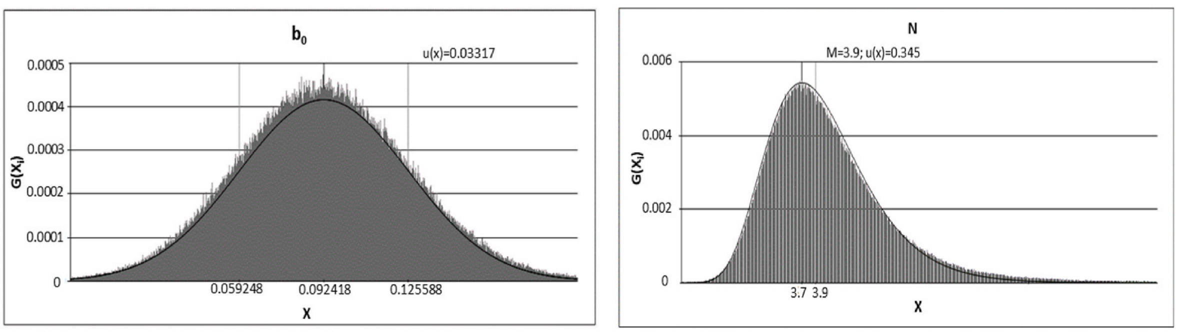

Fig. 5. Resulting probability distribution function for the parameters with 10^6 iterations.

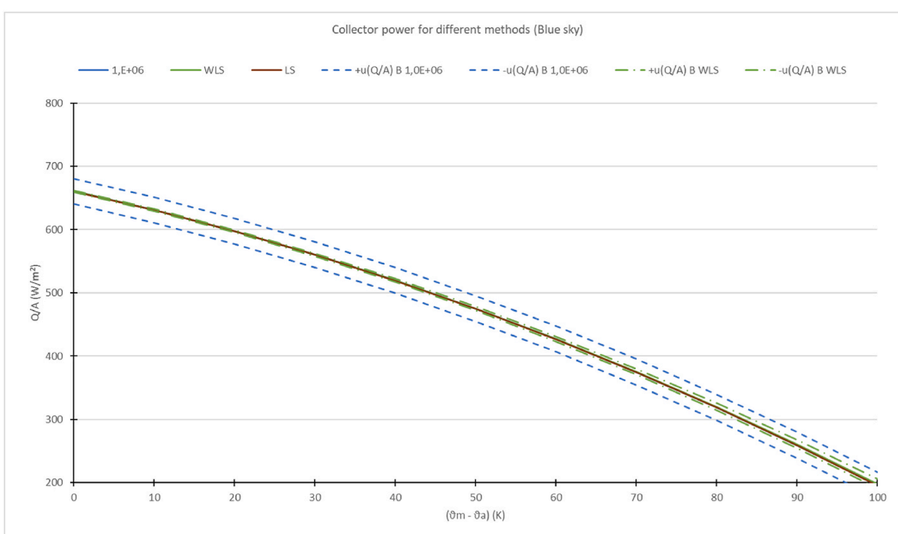

Fig. 6. Flat collector power for the different methods (Blue sky) with the uncertainty limits for 10^6 iterations of the MC_{LS} method and uncertainty limits for WLS method.

4.2.4. *Calculated power*

Fig. 6 shows the calculated power of the flat plate collector for the WLS, LS and the MC_{LS} method for the *Blue sky* condition defined in section 24.3 of ISO 9806:2017. Since the calculated power varies little with the number of iterations only the results for 10^6 iterations are presented.

Table 6
Computation times for the biaxial collector with the MC method.

Iterations	10^3	10^4	10^5	10^6
Time (s)	28.6	280.3	2880.0	28771.5

Little differences exist between methods. Greater differences are observed for the calculated uncertainties between the WLS method and the MC_{LS} method as shown in the figure. For the MC_{LS} method, $u(Q/A_G)$ remains constant over all temperature differences and is greater than the uncertainty calculated for the WLS method (with increase with the temperature difference). This is due to the greater parameter uncertainties obtained for the MC_{LS} method, mainly $u(\eta_0)$. However, $u(Q/A_G)$ for the MC_{LS} method is just a little above the interval calculated for a

Table 7
Mandatory parameters (and their uncertainties) calculated with the MC_{LS} method for the evacuated tubular collector.

Iterations:	10^3	10^4	10^5	10^6	*WLS*	*LS*
η_0	0,641	0,642	0,642	0,641	0,639	0,641
$u(\eta_0)$	0,019	0,020	0,020	0,020	0,004	0,004
b_0	0,100	0,100	0,100	0,100	0,100	0,100
$u(b_0)$	0,000	0,000	0,000	0,000	0,000	0,000
K_d:	1,06	1,05	1,05	1,06	1,06	1,06
$u(K_d)$	0,054	0,054	0,055	0,055	0,019	0,019
a_1	1,54	1,53	1,53	1,53	1,49	1,53
$u(a_1)$	0,07	0,07	0,07	0,07	0,17	0,16
a_2	0,006	0,006	0,006	0,006	0,006	0,006
$u(a_2)$	0,0008	0,0008	0,0008	0,0008	0,0024	0,0022
a_5	14067	14064	14065	14066	15119	14067
$u(a_5)$	255	258	256	255	602	575
$u(Q/A_G)$:	22,473	22,453	22,457	22,459	3417	22,356
Time (s):	29,30	291,76	2944,86	29365,50	4369	2215

Table 8
Mandatory parameters (and their uncertainties) calculated with the MC_{MPFit} method.

Iterations:	10^3	10^4	10^5	10^6	$MPFit_{WLS}$	$MPFit_{LS}$
η_0	0,640	0,639	0,639	0,639	0,639	0,639
$u(\eta_0)$	0,018	0,020	0,020	0,020	0,004	0,004
N	3900	3900	3900	3900	3900	3900
$u(N)$	0,000	0,000	0,000	0,000	0,000	0,000
K_d:	1,06	1,06	1,06	1,06	1,06	1,06
$u(K_d)$	0,053	0,054	0,055	0,055	0,019	0,019
a_1	1,52	1,53	1,53	1,53	1,52	1,53
$u(a_1)$	0,07	0,07	0,07	0,07	0,16	0,16
a_2	0,006	0,006	0,006	0,006	0,006	0,006
$u(a_2)$	0,0008	0,0008	0,0008	0,0008	0,0023	0,0023
a_5	14079	14090	14087	14087	14196	14087
$u(a_5)$	253	259	257	257	580	577
$u(Q/A_G)$:	22,509	22,546	22,536	22,535	6536	22,430
Time (s):	28,80	286,40	2879,80	29510,28	0,113	0,156

level of confidence of $p = 95\ \%$, presented in section 5.1.2.

A similar graph is achieved for the MC_{MPFit} method and for the MC_{LS} method using the 30 s average for the *Blue sky* condition. Compared with the graph in Fig. 6, the calculated power for the MC_{MPFit} method has lower values in the entire range (between 2.6 W/m^2 and 4.7 W/m^2) and the calculated power for the MC_{LS} method using the 30 s average has no significant differences. This is also true for the LS and WLS methods between the 30 s and 300 s averages.

4.2.5. *Quasi-dynamic test results of an evacuated tubular collector with the MC method*

The MC_{LS} method was used to compute the mandatory parameters (and their uncertainties) for the evacuated tubular collector with biaxial geometry presented in 2.1. The computations for this collector demand more computational power due to the extra parameters needed for the dummy variables approach [15]. Table 6 shows the computation times as a function of iterations for the MC_{LS} method for the biaxial collector. With 10^6 the time is around 8 h. The estimated time for 10^7 iterations is about 80 h.

Table 7 shows the significant parameters (and their uncertainties) as a function of the number of iterations, calculated with the parallel MC_{LS} method. The figure also shows the results achieved with the WLS and LS to compare with.

It's clear from the table that MC_{LS} method values approach the LS values while greater differences exist with the WLS values, comparing with the flat plate collector.

Table 8 shows the mandatory parameters (and their uncertainties) calculated with the MC method (MC_{MPFit}). Note that by referring MPFit method we mean that the longitudinal component, $K_b(0, \theta_l)$, in equation (6) is calculated using equation (7) which is done using the nonlinear method but parameters are estimated by using a multilinear regression method. The subscript in collum title indicates the use of the WLS method or the LS method. Little differences exist among results except for $u(Q/A_G)$, which remains smaller when using the WLS method, but is greater than that obtained in Table 7.

Fig. 7 shows the calculated power of the evacuated tubular collector for the WLS, LS, MPFit and 10^6 iterations of the MC_{LS} method, for the *Blue sky* condition. There is a difference in the power between the MC_{LS} and the WLS/LS methods, that decreases with the temperature difference (between $+7.3$ W/m^2 to -2.8 W/m^2). Greater differences are observed for the calculated uncertainties between the MC_{LS} and the WLS/LS methods, that decrease with higher temperature difference. For the MC_{LS} method, $u(Q/A_G)$ remains constant over all temperature differences and is greater than the uncertainty calculated for the WLS method (witch increase with the temperature difference). This is due to the greater parameter uncertainties obtained for the MC_{LS} method, mainly $u(\eta_0)$. However, $u(Q/A_G)$ for the MC_{LS} method is within the interval calculated for a level of confidence of $p = 95\ \%$, which is given by:

$$\frac{\dot{Q}}{A_G} \pm 1.96 \times u\left(\frac{\dot{Q}}{A_G}\right) \mathrm{W}/m^2 = \frac{\dot{Q}}{A_G} \pm 1.96 \times 22.65 \approx \frac{\dot{Q}}{A_G} \pm 44.4\ \mathrm{W}/m^2$$

Table 9, shows the differences in percent between MC_{LS} and WLS methods. The minus sign indicates a smaller value for the MC_{LS} method.

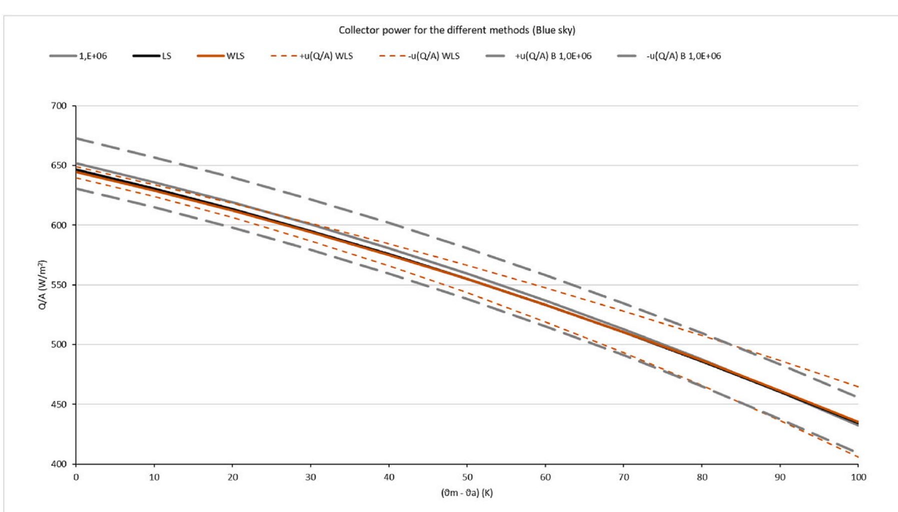

Fig. 7. Evacuated tubular collector power and uncertainty limits for the MC_{LS}, WLS and LS methods (Blue sky).

Table 9
Differences in percent for parameters and their uncertainties, between WLS and MC methods.

Iter.	η_0	K_d	a_1	a_2	a_5	u (η_0)	u (K_d)	u (a_1)	u (a_2)	u (a_5)	u(Q/A_G)
10^3	1.02	−0.54	0.11	14.18	−6.68	78.23	64.06	−130.46	−206.33	−131.54	84.89
10^4	1.13	−0.34	0.25	13.78	−6.63	78.10	64.07	−136.91	−216.13	−133.96	84.91
10^5	1.17	−0.40	0.26	13.81	−6.63	78.36	64.34	−135.39	−213.21	−135.02	84.91
10^6	1.18	−0.41	0.27	13.78	−6.62	78.37	64.46	−133.46	−210.34	−133.94	84.91

Table 10
Count of the optimum values of b_0 parameter achieved with 10^4 iterations of the MC method.

b_0	0.1	0.11	0.12	0.19
N°	9920	80	1	1

As can be seen these differences are significant for the uncertainties achieved with the MC_{LS} method, which explains the greater (and constant) calculated uncertainty limits for the collector power.

One problem with the computation of parameters for the evacuated tubular collector with biaxial geometry is the computation of the optimum value for the b_0 parameter based on the *dummy variables* approach. In the results presented the b_0 parameter was computed before entering the MC method iterations and that same value was used during each MC method iteration. This approach is much less time consuming than to compute the optimum value for the b_0 parameter in each MC method iteration. The computation time for the later approach with 10^4 iterations is 7901s against the 280.3 s from Table 6.

Table 10 shows the optimum values of b_0 parameter and their count achieved with 10^4 iterations of the MC method. As can be seen b_0 is 0.1 for 99.2 % of iterations. The computed average for the b_0 parameter gives 0.1 and the differences between the achieved parameters values with the ones presented in Table 7, are not significant.

This option validates the choice made of using the same b_0 for all MC method iterations. The same study was made for the *N* parameter by using equation (7) and its optimum gave 3.9 in all interactions.

5. Conclusions

Our aim in using MC method would be to obtain results for parameters and mainly for their uncertainties closer to those obtained with WLS with the increase in the number of iterations. However, this was not verified, and besides the obtained parameter values are closer, the parameter uncertainties obtained are closer to those obtained with the LS method and with the MPFit method. For biaxial collectors, the differences with the WLS method for the parameter uncertainties, are higher than for a flat plate collector.

The results show little changes in the parameter values and their uncertainties for iterations of the MC method above 10^4. It can be observed that good results are achieved with $10^4/10^5$ MC iterations, and it seems that there is no advantage in using more iterations. However, it is recommended to use at least 10^6 MC iterations to estimate a 95 % coverage interval.

The MPFit method is very useful to calculate the parameters using the IAM model proposed by the standard ISO 9806:2017. It can also be used to calculate the parameters using the IAM model proposed by Ref. [16], given very similar results to the ones achieved with the LS method and not much different from the achieved with WLS method. Another advantage of MPFit are the lower execution times comparing with the LS/WLS methods. This is important factor when using the MC method.

We also conclude with this work that very little differences exist in the collector power curves by using the MC methods presented and the WLS, LS or the MPFit methods. It was visible that the uncertainty for the collector power calculated with the MC methods is greater than the achieved with the LS, WLS or the MPFit methods and remains practically constant over the entire range of the temperature difference, besides the greater values for $u(Q/A_G)$ achieved with the LS and the MPFit methods compared with the WLS method. This is due to the greater parameter uncertainties obtained with the MC methods.

An advantage of the MC method is to obtain the values of the parameters and their corresponding uncertainty, avoiding calculating the first order derivatives to evaluate the respective uncertainties. For more complex collectors this is valuable. Besides it allows to analyse parameter results and uncertainties using different probability distributions and different limits for the maximum standard uncertainty, which can also be useful. A disadvantage of the MC method is that the computation time depends on the number of samples in the experimental test quantity files. For the average interval of 30 s, the computation time could be 10 times greater than with average interval of 300 s. This problem is worse for evacuated tubular collectors due to the extra parameters used in applying the method introduced by Ref. [14] in the use of equation (6). However, as can be seen in practice iterations above 10^5 do not present significant differences. For QDT testing of collectors we think that the advantages of the MC method outweigh the disadvantages. To be more precise, in this evaluation, we need to make additional studies for other collectors which is a goal for future work.

CRediT authorship contribution statement

João Carlos Rodrigues: Writing – original draft, Validation, Software, Investigation. **Jorge Facão:** Writing – review & editing, Validation. **Maria João Carvalho:** Writing – review & editing, Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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